E. VILLAMOR AND F. XIAO
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ON THE BRENNAN CONJECTURE

ENRIQUE VILLAMOR AND FENG XIAO

Abstract

In this paper we study the conformal mappings of some symmetric simply connected domains in the complex plane whose boundary are fractal trees. In particular we present, based on the alternative simplified approach of Carleson and Makarov to the Brennan conjecture, see [CM], some evidence towards the truth of this conjecture.

1. Introduction

Let Ω be a simply connected domain with at least two boundary points in the extended complex plane, and let Φ be a conformal mapping of Ω onto the open unit disk. Is

$$\iint_{\Omega}|\Phi'|^{p}dxdy<\infty$$

for $4/3<p<4$?

If $p=2$ this integral represents the area of the unit disk and therefore is finite. If Ω is the plane slit along the negative real axis, then the integral converges for $4/3<p<4$ and diverges for $p=4/3$ and $p=4$. This follows easily from a direct computation.

In [Br], Brennan proved that there exists a constant $\tau>0$ which does not depend on Ω such that

$$\iint_{\Omega}|\Phi'|^{p}dxdy<\infty$$

for $4/3<p<3+\tau$.

He also proved that for a large class of domains, including starlike and close to convex domains, $p=4$ is the correct upper bound.

In [Po], Pommerenke proved that

$$\iint_{\Omega}|\Phi'|^{p}dxdy<\infty$$

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