

THE ZERO, POLE AND ORDER OF MEROMORPHIC
SOLUTIONS OF DIFFERENTIAL EQUATIONS
WITH MEROMORPHIC COEFFICIENTS*

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Abstract

In this paper, we investigate the complex oscillation of non-homogeneous linear differential equations with meromorphic coefficients under subtracting the condition that all solutions of differential equation are meromorphic functions.

1. Introduction and results

Consider non-homogeneous linear differential equations of the form

$$(1.1) \quad f^{(k)} + b_{k-1}f^{(k-1)} + \cdots + b_0f = H(z) \quad (k \geq 1)$$

where b_{k-j} ($j=1, \dots, k$) are rational functions, $H(z)$ is a meromorphic function. Z.-X. Chen and S.-A. Gao proved in [3].

THEOREM A. *Let b_{k-j} ($j=1, \dots, k$) be rational functions having a pole at ∞ of order $n_{k-j} \geq 0$, $k \geq 1$, $H(z)$ be a meromorphic function, $\sigma(H) = \beta$ satisfying*

$$(1.2) \quad 1 + \max_{1 \leq j \leq k} n_{k-j}/j < \beta < \infty.$$

If all solutions f of the differential equation (1.1) are meromorphic functions, then

- (a) $\sigma(f) = \beta$.
- (b) $\lambda(1/f) = \lambda(1/H)$, $\tilde{\lambda}(1/f) = \tilde{\lambda}(1/H)$. If $\lambda(H) > \lambda(1/H)$, then $\lambda(f) \geq \lambda(H)$.
- (c) If $\beta > \max\{\lambda(H), \lambda(1/H)\}$, then all solutions of (1.1) satisfy $\tilde{\lambda}(f) = \lambda(f) = \sigma(f) = \beta$, except at most a possible one. The possible exceptional one f_0 satisfies $\lambda(f_0) < \beta$.

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