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MONOTONE DISCONTINUITY OF LATTICE OPERATIONS IN A QUASILINEAR HARMONIC SPACE

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Abstract

We claim, contrary to the linear case, that the lattice operations among harmonic functions are not necessarily monotone continuous in quasilinear harmonic spaces by showing the existence of a quasilinear harmonic space (X, H) in which there are harmonic functions u_n in $H(X)(n=1,2,\cdots,\infty)$ with the following properties: the least harmonic majorant $u_n \vee 0$ and the greatest harmonic minorant $u_n \wedge 0$ of u_n and 0 exist in $H(X)$ for every $n=1,2,\cdots,\infty$; the sequence $(u_n)_{1\leq n<\infty}$ is increasing and convergent to u_∞ on X; the sequence $(u_n\wedge 0)_{1\leq n<\infty}$ converges increasingly to a harmonic function strictly less than $u_\infty \wedge 0$ on X.

1. Introduction

In the theory of $\mathcal{A}$-harmonic functions (including p-harmonic functions) as developed by Heinonen, Kilpeläinen, and Martio in their monograph [2], the order structure and in particular the induced lattice structure of the space of $\mathcal{A}$-harmonic functions (see 5 below) supplement the lack of its linear structure. In this sense the availability of the monotone continuity of lattice operations would greatly enrich the $\mathcal{A}$-harmonic function theory. More specifically, denote by $u\vee v$ ($u\wedge v$, resp.) the least $\mathcal{A}$-harmonic majorant (the greatest $\mathcal{A}$-harmonic minorant, resp.) of two $\mathcal{A}$-harmonic functions u and v on a region Ω of the m-dimensional Euclidean space $\boldsymbol{R}^m$, if it exists. Consider $\mathcal{A}$-harmonic functions u_n $(n=1, 2, \cdots, \infty)$ such that both $u_n\vee 0$ and $u_n \wedge 0$ exist on Ω $(n=1, 2, \cdots, \infty)$. We wish to know whether the following statement is true or not.

2. STATEMENT. *If the sequence $(u_n)_{1\leq n<\infty}$ is increasing and convergent to u_∞ on Ω, then $(u_n\wedge 0)_{1\leq n<\infty}$ converges to $u_\infty\wedge 0$ on Ω:*

$$\lim_{n\to\infty} u_n\wedge 0 = u_\infty \wedge 0. \tag{3}$$

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