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ON THE BIFURCATION SET OF A POLYNOMIAL FUNCTION AND NEWTON BOUNDARY, II

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1. Introduction

1.1. Let $f: \boldsymbol{C}^n \to \boldsymbol{C}$ be a polynomial function and let us denote by B_f the *bifurcation set* of f, i.e. B_f is the smallest subset $\Gamma \subseteq \boldsymbol{C}$ such that the restriction $f: \boldsymbol{C}^n \backslash f^{-1}(\Gamma) \to \boldsymbol{C} \backslash \Gamma$ is a locally trivial fibration. It is well known that B_f is a finite set (see for example [13], [3], [11]) containing not only the set Σ_f of critical values of f, but also some *extra* values, corresponding to the so called "critical points at infinity". The problem of describing the bifurcation set B_f was considered by several authors, see for example: [3], [1], [10], [2], [12], [7]. In this note we would like to prove that certain values, given in [7] as possible elements of B_f, really belong to the bifurcation set of a *Newton nondegenerate* polynomial f.

1.2. We recall now some definitions and notations. Let $f: \boldsymbol{C}^n \to \boldsymbol{C}$ be a polynomial function. We shall assume that $f(0)=0$ If

$$f(z) := \sum_{\nu \in \boldsymbol{N}^n} a_\nu z^\nu,$$

we denote:

$$\operatorname{supp}(f) := \{\nu \in \boldsymbol{N}^n \mid a_\nu \neq 0\} \subseteq \boldsymbol{R}^n,$$

$$\overline{\operatorname{supp}(f)} := \text{the convex closure in } \boldsymbol{R}^n \text{ of } \operatorname{supp}(f),$$

$$\tilde{\Gamma}_-(f) := \text{the convex closure in } \boldsymbol{R}^n \text{ of } \{0\} \cup \operatorname{supp}(f).$$

For $\Delta \subseteq \boldsymbol{R}^n$ we put

$$f_\Delta := \sum_{\nu \in \Delta} a_\nu z^\nu$$

and we say that f is *nondegenerate* on Δ if the system of equations

$$\frac{\partial f_\Delta}{\partial z_1}(z) = \cdots = \frac{\partial f_\Delta}{\partial z_n}(z) = 0$$

has no solutions in $(\boldsymbol{C} \backslash \{0\})^n$. We say that f is *Newton nondegenerate* if for every compact face Δ of $\tilde{\Gamma}_-(f)$, with $0 \notin \Delta$, we have that f is nondegenerate

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