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A NOTE ON THE POINCARÉ-BENDIXSON INDEX THEOREM

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Abstract

The local scheme for an equilibrium state of an analytic planar dynamical systems is investigated. Upper bounds of the numbers of elliptic and hyperbolic sectors are derived. Methods of singularity theory are applied to obtain appropriate estimations in terms of indices of maps explicitly constructed from a vector field.

I. Introduction

The study of geometric differential equations was founded by H. Poincaré in his classical "Mémoire" [PCR1] (see also [PCR2], [PCR3]).

At 15 years distance, Poincaré's ideas was followed by Bendixson's whose attention was mainly turned to the local phase-portrait around a critical point. In his major paper [BDX] Bendixson derived the index formula

$$\deg(F)=1+\frac{\mathcal{E}-\mathcal{H}}{2}$$

where $\deg(F)$ is the index of a stationary point of a planar vector field and $\mathcal{E}$, $\mathcal{H}$ are respectively the numbers of elliptic and hyperbolic sectors. This equality, known in bibliography as the Poincaré-Bendixson formula, gives an interesting application of topological methods to planar differential equations.

Under some additional assumptions one can give another Poincaré-Bendixson formula

$$\deg(F)=1+\frac{n_e-n_h}{2}$$

where n_e, n_h are respectively the numbers of internal and external tangent points of a vector field to a C, Jordan curve going around a stationary point.

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