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KODAI MATH. J.
19 (1996), 62–86

PSEUDOHERMITIAN IMMERSIONS, PSEUDO-EINSTEIN STRUCTURES, AND THE LEE CLASS OF A CR MANIFOLD

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Any nondegenerate CR manifold carrying a fixed contact 1-form is known to possess (cf. N. Tanaka [T], S. Webster [W1]) a canonical linear connection (the *Tanaka-Webster connection*) parallelizing the Levi form and the maximal complex structure. This leads to an (already widely exploited, cf. D. Jerison & J.M. Lee [JL1], [JL2], J.M. Lee [L1], [L2], H. Urakawa [U1], [U2], etc.) analogy between CR geometry on one hand, and both Hermitian and conformal geometry on the other.

To describe our point of view, let M and A be two CR manifolds of CR dimensions n and $N=n+k$, $k\geqq 1$, respectively. A *CR immersion* $f: M\to A$ is an immersion and a CR map. If f is the inclusion then M is a *CR submanifold* of A (a *CR hypersurface* when $k=1$). For instance, let M^{2n+1} be the intersection between the sphere S^{2n+3} and a transverse complex hypersurface in $\boldsymbol{C}^{n+2}$. Then M^{2n+1} is a CR hypersurface of S^{2n+3} (in particular M^{2n+1} is strictly pseudoconvex). Let M be a CR submanifold of A. Then M is *rigid* in A if any CR diffeomorphism $F: M\to M'$ onto another CR submanifold M' of A (e.g. F may be the restriction of a biholomorphic mapping) extends to a CR automorphism of A (e.g. if $A=S^{2n+3}$ then F should extend to a fractional linear, or projective, transformation preserving S^{2n+3}). A theory of CR immersions has been initiated by S. Webster [W2]. There it is shown that S^{2n+1} is rigid in S^{2n+3} if $n\geqq 2$. Also, if $n\geqq 3$ then any CR hypersurface of S^{2n+3} is rigid. The basic idea in [W2] is to endow the ambient space S^{2n+3} with the Tanaka-Webster connection (rather than the Levi-Civita connection associated with the canonical Riemannian structure) and obtain CR analogues of the Gauss-Weingarten (respectively Gauss-Ricci-Codazzi) equations (from the theory of isometric immersions between Riemannian manifolds). In the end, these could be used to show that the intrinsic geometry determines the (CR analogue of the) second fundamental form of the given CR immersion. The main inconvenience of this approach seems to be the nonuniqueness of choice of a canonical connection on the CR submanifold (i.e. the induced and the 'intrinsic' Tanaka-Webster connections of the submanifold do not coincide, in general). In [D1] we compensate

Received September 28, 1994, revised April 7, 1995.