X. HUANG
KODAI MATH. J.
19 (1996), 17—25

ON GEHRING-POMMERENKE'S ESTIMATE FOR QUASICIRCLES

XINZHONG HUANG

1. Introduction

A quasicircle can be regarded as a point of the universal Teichmüller space, which can be considered also as a set of Schwarzian derivatives by Bers' embedding theorem. In particular, we know that, if the norm of a Schwarzian derivative is less than 2, then the derivative corresponds to a quasicircle (see, for example [11], [2], [3], [4]).

Furthermore, Gehring and Pommerenke [7] found that, if the norm of a Schwarzian derivative is less than 2, then the complexity of the corresponding quasicircle can be estimated by the norm. The main result of this paper improves the estimate due to Gehring and Pommerenke.

We recall that, in a domain A, the Schwarzian derivative of f is defined as

$$S_f(z)=\left(\frac{f''}{f'}\right)'-\frac{1}{2}\left(\frac{f''}{f'}\right)^2, \tag{1.1}$$

where $f(z)$ is meromorphic and locally univalent in A.

We see that S_f is analytic in A and satisfies

$$S_{\varphi\circ f\circ\psi}(z)=S_f(\psi(z))\psi'^2(z)+S_\psi(z) \tag{1.2}$$

for any conformal mapping ψ of another domain A' onto A and $\varphi\in$Möb, where Möb denotes the group of Möbius transformations.

Let A be a simply connected domain conformally equivalent to a disk. The norm of Schwarzian derivative of f in A is defined as

$$\|S_f\|_A=\sup_{z\in A}|S_f(z)|\lambda^{-2}(z), \tag{1.3}$$

where λ is the Poincaré density on A.

Next, by the transformation rule (1.2) and conformal invariance of the hyperbolic metric, we have

$$\|S_f\|_A=\|S_{f\circ\psi}-S_\psi\|_{A'}. \tag{1.4}$$

1980 Mathematics subject classifications: Primary 30C60; Secondary 30C75.
Received June 2, 1994; received January 17, 1995.