P. LI AND C.-C. YANG
KODAI MATH J.
18 (1995), 437—450

SOME FURTHER RESULTS ON THE UNIQUE RANGE SETS OF MEROMORPHIC FUNCTIONS

PING LI AND CHUNG-CHUN YANG*

Abstract

By improving a generalization of Borel's theorem, the authors have been able to show that there exists a finite set S with 15 elements such that for any two nonconstant meromorphic functions f and g the condition $E_f(S)=E_g(S)$ implies $f\equiv g$. As a special case this also answers an open question posed by Gross [1] about entire functions, and has improved some results obtained recently by Yi [10]. In the last section, the uniqueness polynomials of meromorphic functions which is related to the unique range sets has been studied. A necessary and sufficient condition for a polynomial of degree 4 to be a uniqueness polynomial is obtained.

1. Introduction

Let f be a nonconstant meromorphic function on the complex plane C and S be a subset of distinct elements in C. Define

$$E_f(S)=\bigcup_{a\in S}\{z\,|\,f(z)-a=0\},$$

here a zero of $f(z)-a$ of multiplicity m appears m times in $E_f(S)$. Usually, the notation $\bar{E}_f(S)$ express the set which contains the same points as $E_f(S)$ but without counting multiplicities. About sixty years ago, R. Nevanlinna [6] proved two general results: (1). If two nonconstant meromorphic functions f and g satisfy $\bar{E}_f(a_i)=\bar{E}_g(a_i)$ $(i=1, \cdots, 5)$ where a_i $(i=1, \cdots, 5)$ are distinct points in $\bar{C}$, then $f\equiv g$. (2) If two nonconstant meromorphic functions f and g satisfy $E_f(a_i)=E_g(a_i)$ $(i=1, \cdots, 4)$ where a_i $(i=1, \cdots, 4)$ are distinct points in $\bar{C}$, then f is a Möbius transformation of g. Actually, above notations S and $E_f(S)$ can be regarded as a range set and a preimage set of f respectively. Recent years, in several papers, for examples [1], [2], [4], [7] and [10], properties of range set and preimage set which can, to some extent, uniquely determine the mero-

* The research was partially supported by a UPGC grant of Hong Kong
Keywords: meromorphic function, entire function, unique range set, uniqueness polynomial.
1991 Mathematics Subject Classification. Primary 30D35.
Received June 24, 1994; revised December 1, 1994.