S. HU AND N.S. PAPAGEORGIOU
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ON NONLINEAR, NONCONVEX EVOLUTION INCLUSIONS

SHOUCHUAN HU AND NIKOLAOS S. PAPAGEORGIOU*

Abstract

We consider a nonlinear evolution inclusion driven by an m-accretive operator which generates an equicontinuous nonlinear semigroup of contractions. We establish the existence of extremal integral solutions and we show that they form a dense, G_δ-subset of the solution set of the original Cauchy problem. As an application, we obtain "bang-bang" type theorems for two nonlinear parabolic distributed parameter control systems.

1. Introduction

Let $T=[0, b]$ and X a separable reflexive Banach space, whose dual X^* is uniformly convex. We consider the following multivalued Cauchy problems:

$$(1) \qquad \left\{ \begin{array}{c} -\dot{x}(t) \in Ax(t)+F(t, x(t)) \\ x(0)=x_0 \end{array} \right\},$$

$$(2) \qquad \text{and} \left\{ \begin{array}{c} -\dot{x}(t) \in Ax(t)+\operatorname{ext} F(t, x(t)) \\ x(0)=x_0 \end{array} \right\}.$$

Here $A: D \subseteq X \to 2^X \setminus \{\emptyset\}$ is an m-accretive operator, $F: T \times X \to 2^X \setminus \{\emptyset\}$ is a multifunction and $\operatorname{ext} F(t, x)$ denotes the extreme points of the set $F(t, x)$. By a solution of (1) (resp. of (2)), we mean a function $x(\cdot) \in C(T, X)$ which is an integral solution in the sense of Benilan (see section 2) of the Cauchy problem

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