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SURFACES WITH PARALLEL MEAN CURVATURE VECTOR IN $P^2(C)$

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1. Introduction

The set of surfaces with parallel mean curvature vector in Riemannian manifold, which includes all minimal surfaces in the manifold, has been studied by many geometers. Especially, Chen [1] and Yau [7] studied them in the case that the ambient space is an n-dimensional real space form $\bar{M}^n(c)$ of constant sectional curvature c. They proved that if $x: M \to \bar{M}^n(c)$ is an isometric immersion with parallel mean curvature vector of a two-dimensional Riemannian manifold M into $\bar{M}^n(c)$, then $x(M)$ is one of the following surfaces: (1) *a minimal surface in* $\bar{M}^n(c)$, (2) *a minimal surface of a small hypersphere of* $\bar{M}^n(c)$, and (3) *a surface with constant mean curvature in a 3-sphere of* $\bar{M}^n(c)$. This shows that the study of surfaces in $\bar{M}^n(c)$ with parallel mean curvature vector is reduced to that of minimal surfaces except the case (3).

On the other hand, concerning the surfaces with parallel mean curvature vector in a complex space form, we know several minimal surfaces in the n-dimensional complex projective space $P^n(C)$ with the Fubini-Study metric of constant holomorphic sectional curvature 4ρ. Moreover, many results characterizing them have been obtained (cf. [2], [3], [4], [5], [6]). However, when we concern with non-minimal surfaces in $P^n(C)$ with parallel mean curvature vector, not many such examples are known so far, even for $n=2$.

In Sections 1 and 2 of the previous paper [5], we developed a local theory of surfaces in $P^n(C)$ by using the Kaehler function. By applying it, in this paper we shall study non-minimal immersions $x: M \to P^2(C)$ with parallel mean curvature vector. In fact, in Section 2 we obtain basic formulas for such surfaces in a 2-dimensional Kaehler manifold of constant holomorphic sectional curvature 4ρ. Then, in Sections 3 and 4, we show a method of the local construction of such immersions. Finally, in Section 5 we determine isometric immersions with parallel mean curvature vector field of a Riemannian 2-manifold with constant Gaussian curvature into $P^2(C)$. Theorem 5.2 generalizes a theorem by Ludden, Okumura and Yano [4].

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