H. FUJIMOTO
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NEVANLINNA THEORY FOR MINIMAL SURFACES OF PARABOLIC TYPE

HIROTAKA FUJIMOTO

§ 1. Introduction

Consider a minimal surface $x=(x_1, \cdots, x_m): M \to \boldsymbol{R}^m$ in $\boldsymbol{R}^m$. In the previous papers [9], [10], [11] and [12], the author gave some value-distribution-theoretic properties of the Gauss map of M in the case where M is complete (cf., [13]). On the other hand, under some assumptions, E. F. Beckenbach and his collaborators showed that the map x itself has many properties which are similar to the results in Nevanlinna theory for meromorphic functions on $\boldsymbol{C}$ in their papers [4], [3], [2] and [6]. They developed their theory for 'meromorphic minimal surfaces'. Roughly speaking, these are minimal surfaces in $\boldsymbol{R}^m$ with at worst pole-like singularities which is conformally isomorphic with the complex plane. The purpose of this paper is to extend some of their results to the case where M is conformally isomorphic with a Riemann surface of parabolic type. For brevity, we restrict ourselves to the case of regular minimal surfaces though our arguments are also available for minimal surfaces with pole-like singularities.

By definition, a Riemann surface M is of parabolic type if there is a proper map $\tau: M \to [0, +\infty)$ of class C^∞ such that $dd^c \log\tau=0$ and $dd^c\tau \not\equiv 0$ on $M-M_s$ for some $s>0$, where $M_s:=\{a \in M;\ \tau(a)<s\}$. We define the hyperspherical function by

$$m^0(r;\ c,\ M):=\frac{2}{r}\int_{\partial M_r}\log\frac{1}{|x,\ c|}d^c\tau-\frac{2}{s}\int_{\partial M_s}\log\frac{1}{|x,\ c|}d^c\tau,\ (c\in\bar{\boldsymbol{R}}^m)$$

and the order function for M by $T^0(r;M):=m^0(r;\infty, M)$, where $|x, c|$ denotes a half of the chordal distance between $\varpi^{-1}(x)$ and $\varpi^{-1}(c)$ for the stereographic projection ϖ of the unit sphere in $\boldsymbol{R}^{m+1}$ onto $\bar{\boldsymbol{R}}^m:=\boldsymbol{R}^m \cup \{\infty\}$. We define also the counting function and the visibility function by

$$N(r;\ c,\ M):=\int_s^r \sum_{a\in M_t}\nu_{|x-c|}(a)\frac{dt}{t},$$

$$H(r;\ c,\ M):=\int_s^r\frac{dt}{t}\int_{M_t}dd^c\log|x-c|^2$$

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