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A SHORT ANALYTIC PROOF OF CLOSEDNESS OF LOGARITHMIC FORMS

Dedicated to Professor Nobuyuki Suita on the occasion of his 60th birthday

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§ 1. Introduction

Deligne [D, (3.2.14)] proved the d-closedness of logarithmic forms on a smooth complex quasi-projective variety by showing the degeneracy of some spectral sequence. Actually, his proof works for a Zariski open subspace of a compact Kähler manifold. The logarithmic forms play important roles in various aspects of analytic-algebraic geometry, including the value distribution of holomorphic mappings (see, e. g., [D], [I], [N1], [N2], [N3] and [N4]), and their closedness is fundamental. Therefore it may be of worth, and hence our purpose of this note to give its short proof based only on the classical harmonic integral theory [K].

Let D be an effective reduced divisor on a compact Kähler manifold M, and $\Omega_M^p(\log D)$ the sheaf of germs of logarithmic p-forms along D over M (see § 2 for the definition).

THEOREM. *Let* $\omega \in H^0(M, \Omega_M^p(\log D))$ *be a global section of* $\Omega_M^p(\log D)$. *Then* $d\omega=0$ *in the complement* $M \setminus D$ *of* D.

§ 2. Definitions and Lemmas

In the present note we denote by M an m-dimensional compact Kähler manifold with structure sheaf $\mathcal{O}_M$ of germs of holomorphic functions. Let Ω_M^p denote the sheaf of germs of holomorphic p-forms over M. Let D be an effective reduced divisor on M. Let $x \in M$ and take irreducible $\sigma_j \in \mathcal{O}_{M,x}$, $1 \leqq j \leqq k$, so that $\{\sigma_j=0\}$ define the local irreducible components of D at x. Then we define the sheaf $\Omega_M^1(\log D)$ of germs of logarithmic 1 forms along D by

$$\Omega_{M,x}^1(\log D)=\sum_{j=1}^{k} \mathcal{O}_{M,x}\frac{d\sigma_j}{\sigma_j}+\Omega_{M,x}^1 .$$

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