M. TANABE
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AUTOMORPHISMS OF RAMIFIED COVERINGS OF RIEMANN SURFACES

MASAHARU TANABE

1. Introduction

Let M, N be Riemann surfaces, $\pi: M \to N$ be an unlimited covering map, and $T \in \mathrm{Aut}(M)$. The purpose of this paper is to study the existence of some $T_* \in \mathrm{Aut}(N)$ which makes the following diagram commutative.

$$\begin{array}{ccc} M & \xrightarrow{T} & M \\ \downarrow \pi & T_* & \downarrow \pi \\ N & \longrightarrow & N \end{array}$$

In general, it is not always the case that there exists a $T_* \in \mathrm{Aut}(N)$ which satisfies $\pi \circ T = T_* \circ \pi$. We will exhibit in section 3 an example such that there is no $T_* \in \mathrm{Aut}(N)$ which satisfies $\pi \circ T = T_* \circ \pi$. There are few papers concerning this problem, and the author can find only a paper by Martens [5] which gives a necessary and sufficient condition for the existence of $T_* \in \mathrm{Aut}(N)$ when M, N are compact Riemann surfaces in terms of homology groups. In this paper, we do not assume that Riemann surfaces M, N are compact. We assume that a covering map $\pi: M \to N$ is unlimited, ramified, finite-sheeted, and $T \in \mathrm{Aut}(M)$ fixes all of the ramification points of π. We say that (M, π, N) is of excluded type if $N = \boldsymbol{C}$ and the image set of ramification points of π consists of one point on N. We shall show

THEOREM. *Let $\pi: M \to N$ be an unlimited, ramified, finite-sheeted covering, and of non-excluded type. Then there exist a Riemann surface S and unlimited covering maps $\sigma: M \to S$ and $q: S \to N$ such that $\pi = q \circ \sigma$, where q is unramified, and for any $T \in \mathrm{Aut}(M)$ fixing all of the ramification points of π there exists a $T_p \in \mathrm{Aut}(S)$ making the following diagram commutative.*

$$\begin{array}{ccc} M & \xrightarrow{T} & M \\ \downarrow \sigma & T_p & \downarrow \sigma \\ S & \longrightarrow & S \end{array}$$

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