T. H. KIM AND E. S. KIM
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ITERATIVE FIXED POINTS OF NON-LIPSCHITZIAN SELF-MAPPINGS

TAE HWA KIM* AND EUN SUK KIM

Abstract

In this paper, we shall establish iterative fixed points of non-Lipschitzian continuous self-mappings on Banach spaces with weak uniform normal structure.

1. Introduction

Let C be a nonempty subset of a real Banach space X and let $\boldsymbol{N}$ be the set of natural numbers. A mapping $T: C \to C$ is said to be Lipschitzian if for each $n \in \boldsymbol{N}$, there exists a real number $k(n)$ such that

$$\|T^n x - T^n y\| \leqq k(n)\|x-y\| \qquad \text{for all} \quad x,\ y \in C.$$

In particular, T is said to be asymptotically nonexpansive [7] if $\lim_{n\to\infty} k(n)=1$ and it is said to be nonexpansive if $k(n)=1$ for any $n \in \boldsymbol{N}$. We now consider a non-Lipschitzian self-mapping on C, that is to say, a mapping of weakly asymptotically nonexpansive type. We say that a mapping $T: C \to C$ is said to be weakly asymptotically nonexpansive type (simply, w.a.n.t.) on C (see [10]) if, for each $x \in C$ and each bounded subset D of C,

$$\limsup_{n\to\infty}(\sup\{[\|T^n x - T^n y\| - \|x-y\|] : y \in D\}) \leqq 0.$$

Immediately, we can see that all mappings of w.a.n.t. include all mappings of asymptotically nonexpansive type (see [11]). In particular, if $T: C \to C$ is a Lipschitzian mapping with an additional condition, i.e., $\limsup_{n\to\infty} k(n) \leqq 1$ (see [12] and [14]), then it is obviously a continuous mapping of w.a.n.t. Further if C is bounded, then any mapping of w.a.n.t. is asymptotically nonexpansive type.

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