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ENTIRE FUNCTIONS THAT SHARE ONE VALUE WITH THEIR DERIVATIVES

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Abstract

The paper generalizes a result of [2] and makes an example which shows that the generalization is precise. Also we get similar conclusions in other cases.

§ 1. Introduction

We say that nonconstant meromorphic functions f and g share the value a provided that $f(z)=a$ if and only if $g(z)=a$. We will state whether the shared value is by CM (counting mulitiplicities) or by IM (ignoring multiplicities).

L. Rubel and C. C. Yang proved the following theorem:

THEOREM A[1]. *Let $f(z)$ be a nonconstant entire function. If f and f' share two distinctive valves a and b IM, then $f \equiv f'$.*

1986, Jank, Mues and Volkman proved:

THEOREM B[2]. *Let $f(z)$ be a nonconstant entire function. If f and f' share the value a $(a \neq 0)$, and $f''(z)=a$ when $f(z)=a$, then $f \equiv f'$.*

It is asked naturally whether the f'' of Theorem B can be simply replaced by $f^{(k)}$ $(k \geqq 3)$. We make an example which shows that the answer of this question is negative.

Let k be a positive integer $(k \geqq 3)$ and let $\omega(\neq 1)$ be a $(k-1)$-th root of unity. Set $g(z)=e^{\omega z}+\omega-1$. It is easy to know that g, g' and $g^{(k)}$ share the value ω CM, but $g \not\equiv g'$ and $g \not\equiv g^{(k)}$.

Between the example and Theorem B we will prove the following results.

THEOREM 1. *Let $f(z)$ be a nonconstant entire function. If f and f' share the value a $(a \neq 0)$ CM, and $f^{(n)}(z)=f^{(n+1)}(z)=a$ $(n \geqq 1)$ when $f(z)=a$, then $f \equiv f^{(n)}$.*

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