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INTERSECTIONS OF MINIMAL SUBMANIFOLDS IN MANIFOLDS OF PARTIALLY POSITIVE CURVATURE

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1. Introduction and main theorems

In 1961, T. Frankel [3] showed that two compact totally geodesic submanifolds P and Q in a Riemannian manifold N of positive sectional curvature must necessarily intersect if their dimension sum is at least that of N. If N is also a Kaehler manifold, he proved a more satisfactory result: There, instead of requiring P and Q to be totally geodesic, under the assumption that they are analytic submanifolds, he obtained the same conclusions under the assumptions. The proof was done by a nice application of the formula for the second variation of arc-length of a geodesic derived by Synge in [11].

Synge once used the second variational formula to prove that an even dimensional orientable compact manifold with positive sectional curvature is simply connected. It turns out that the Synge's method for the second variation of arc-length of a geodesic works as a powerful tool in differential geometry. In the present paper, we shall study the intersections of minimal submanifolds in a Riemannian manifold with partially positive curvature and analytic submanifolds in a Kaehler manifold with partially positive holomorphic bisectional curvature by using this method. We remark that Galloway and Rodriguez [7] studied the same problems for minimal submanifolds in a manifold with non-negative sectional curvature by a different approach but our results have few relations with theirs.

Before stating our theorems, we recall some concepts of partially positive curvature (cf. [12]). Let N^n be an n-dimensional Riemannian manifold and $p \in N^n$ be a point of N^n. If, for any $(k+1)$ mutually orthogonal unit tangent vectors $e, e_1, \cdots, e_k \in T_pN^n$, we have $\sum_{i=1}^{k} K(e\wedge e_i)>0$ (resp. $\geqq 0$), we say N^n has positive (resp. nonnegative) k-th Ricci curvature at p. If N^n has positive (resp. nonnegative) k-th Ricci curvature at every point of it, we call N^n has positive (resp. nonnegative) k-th Ricci curvature and denote this fact by $Ric_{(k)}(N^n)>0$ (resp. $\geqq 0$). Here, $K(e\wedge e_i)$ denotes the sectional curvature of the

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