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NEF LINE BUNDLES ON ALGEBRAIC SURFACES

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§ 0. Introduction

In this paper we assume throughout that the ground field k is algebraically closed and of characteristic $p \geqq 0$.

Let X be a projective surface and L a line bundle on X. When L is *nef* (i.e., $LC \geqq 0$ for all integral curves C on X), the pair (X, L) will be called a *semipolarized surface*. We say that two semipolarized surfaces (X_1, L_1) and (X_2, L_2) are *birationally equivalent* if there is a projective surface W with birational morphisms $f_i: W \to X_i$ $(i=1, 2)$ such that $f_1^* L_1 = f_2^* L_2$. Moreover, if X is normal, the *sectional genus* $g(X, L)$ of the semipolarized normal surface (X, L) is given by the formula $2g(X, L)-2=(\omega_X+L)L$, where ω_X is the canonical sheaf of X.

Lanteri and Palleschi proved the following on the assumption that L is an ample line bundle on a smooth complex projective surface X.

THEOREM 0.1 ([LP], Remark 1.3). *Let L be an ample line bundle on a smooth complex projective surface X. Then one of the following holds.*

(1) *K_X+L is nef for the canonical bundle K_X of X.*
(2) $(X, L) \cong (\boldsymbol{P}^2, \mathcal{O}_P(1))$.
(3) $(X, L) \cong (\boldsymbol{P}^2, \mathcal{O}_P(2))$.
(4) *(X, L) is a scroll over a smooth curve* (For the definition of a scroll, see § 1.).

THEOREM 0.2 ([LP], Remark 1.1 and Corollary 2.3). *Let L be an ample line bundle on a smooth complex projective surface X. Then $g(X, L) \geqq 0$. Moreover, if $g(X, L)=0$, then (X, L) is one of the following.*

(1) $(X, L) \cong (\boldsymbol{P}^2, \mathcal{O}_P(1))$.
(2) $(X, L) \cong (\boldsymbol{P}^2, \mathcal{O}_P(2))$.
(3) *(X, L) is a scroll over $\boldsymbol{P}^1$.*

THEOREM 0.3 ([LP], Corollary 2.4). *Let L be an ample line bundle on a smooth complex projective surface X. If $g(X, L)=1$, then (X, L) is one of the following.*

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