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A CONTINUOUS VERSION OF THE RELAXATION THEOREM FOR NONLINEAR EVOLUTION INCLUSIONS

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Abstract

We consider parametric nonlinear evolution inclusions defined on an evolution triple of spaces. First we prove some continuous dependence results for the solution sets of both the convex and nonconvex problem and for the set of solution-selector pairs of the convex problem. Subsequently, we derive a parametrized version of the Filippov-Gronwall estimate in which the parameter varies in a continuous fashion. Using that estimate, we prove a continuous version of the nonlinear relaxation theorem. An example of a nonlinear parabolic control system is worked out in detail.

1. Introduction

One of the fundamental results in the theory of differential inclusions (set-valued differential equations), is the "relaxation theorem". It says that if the orientor field (set-valued vector field) is h-Lipschitz in the state variable, then the solution set of the differential inclusion is dense in that of the convexified problem (i.e. the system obtained by replacing the original vector field by its closed, convex hull). We refer to the book of Aubin-Cellina [2] (theorem 2, p. 124) which treats differential inclusions in $\boldsymbol{R}^N$ and the papers of Papageorgiou [15] and Zhu [24], which deal with differential inclusions in Banach spaces. Such a density result is important in control theory because it leads to nonlinear versions of the "bang-bang" principle. Recently, the relaxation theorem was extended to evolution inclusions by Frankowska [6] and Papageorgiou [20] for semilinear systems, by Papageorgiou [17] for integrodifferential systems and by Hu-Lakshmikantham-Papageorgiou [9] for nonlinear systems. Furthermore in [16], we studied the relation between relaxability and performance stability for nonlinear variational problems monitored by nonlinear evolution equations. We

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