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ON THE SPECTRUM OF HOMOGENEOUS SPHERICAL SPACE FORMS

Dedicated to Professor Hideki Ozeki on the occasion of his sixtieth birthday

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1. Introduction

Let S^{2n-1} $(n>1)$ be the $(2n-1)$-dimensional sphere of constant curvature 1 and G be a finite subgroup of the orthogonal group of degree $2n$ acting fixed point freely on S^{2n-1}. Then the spherical space form $M=S^{2n-1}/G$ has a constant curvature 1. If G is cyclic then the spherical space form is called a lens space. The Laplacian Δ acting on the space of smooth functions on a spherical space form has a discrete spectrum with finite multiplicities. It is well known that the eigenvalues of the Laplacian Δ are of the form $k(k+2n-2)$ $(k=0, 1, 2, \cdots)$. Let a_k be the multiplicity of eigenvalue $k(k+2n-2)$. The Poincaré series $\sum_{k=0}^{\infty} a_k z^k$ associated to the spectrum for homogeneous spherical space becomes a rational function which is a nice form to study the spectrum of spherical space forms and has been studied by the author in order to construct riemannian manifolds which are isospectral but not isometric. On the other hand, the classification for homogeneous spherical space forms is given by Wolf [4]. Wolf's classification theorem for homogeneous space forms states that for any $g\in G$ of a homogeneous spherical space form S^{2n-1}/G, there is a unimodular complex number λ such that half the eigenvalues of g are λ and the other half are $\bar{\lambda}$. By this theorem, the Poincaré series associated to the spectrum for homogeneous spherical space forms becomes a simple form.

In this paper using this form we give the spectrum of homogeneous spherical space forms explicitly. For the cases of the spheres and homogeneous lens spaces their spectra are given in M. Berger [1] and in T. Sakai [3]. Our table for the spectrum of homogeneous lens spaces is different from Sakai [3], and is more simpler than Sakai's table.

2. Preliminary

In this section we give elementary formulae which we need in the following sections. The following two lemmas are well known.

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