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KODAI MATH. J.
18 (1995), 44—56

ON SINGULAR SOLUTIONS FOR A SEMILINEAR ELLIPTIC EQUATION

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1. Introduction

Let Ω be a bounded domain in $\boldsymbol{R}^n$ $(n\geqq 2)$ with smooth boundary $\partial\Omega$. And let Σ be a C^∞-compact submanifold of Ω of dimension m $(0\leqq m\leqq n-1)$. We take an arbitrary $\alpha(x)\in C^\infty(\Sigma)$ such that $\alpha(x)>0$ on Σ and consider the following equation.

$$\begin{cases} -\Delta u=u^p+\alpha\delta_\Sigma & \text{in } \mathcal{D}'(\Omega) \quad (p>1) \\ 0\leqq u\in C^2(\Omega\setminus\Sigma), & \end{cases} \tag{1.1}$$

where δ_Σ is the measure defined by

$$\langle\delta_\Sigma,\ \eta\rangle=\int_\Sigma \eta(\sigma)d\sigma \tag{1.2}$$

for any $\eta\in C_0^\infty(\Omega)$.

What can one say about the existence of a solution of (1.1) and the local behaviour of its solution near Σ? We have the following.

THEOREM 1. *There exists a solution of* (1.1) *if and only if* $1<p<(n-m)/(n-m-2)$ $(1<p<\infty$ *if* $n-m\leqq 2)$. *And there exists a solution u of* (1.1) *satisfying*

$$\begin{cases} C_1 d(x)^{-(n-m-2)}\leqq u(x)\leqq C_2 d(x)^{-(n-m-2)} & \textit{near } \Sigma \ (\textit{if } m\leqq n-3) \\ C_1|\log d(x)|\leqq u(x)\leqq C_2|\log d(x)| & \textit{near } \Sigma \ (\textit{if } m=n-2) \\ u(x)\in C^0(\Omega) \quad (\textit{if } m=n-1), & \end{cases} \tag{1.3}$$

where $d(x)$ denotes the distance between x and Σ. Here C_1 and C_2 denote some positive constants.

THEOREM 2. *Assume that* $p<n/(n-2)$ $(p<\infty$ *if* $n=2)$. *Then the same bounds as in* (1.3) *hold for any u satisfying* (1.1) *and, in addition,*

$$u\in L^p_{\mathrm{loc}}(\Omega) \qquad \textit{if } m=n-1\,. \tag{1.4}$$

Received May 2, 1992; revised February 21, 1994.