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HAYMAN DIRECTION OF MEROMORPHIC FUNCTIONS

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Abstract

Let f be meromorphic in the plane. Then f has a Hayman direction provided that

$$\limsup_{r\to\infty}\frac{T(r,f)}{(\log r)^2}=\infty .$$

1. Introduction

We define a Hayman direction of a meromorphic function $f(z)$ to be a ray $\arg z=\theta$, $0\leqq\theta\leqq 2\pi$, such that for every positive integer l and positive $\varepsilon>0$,

$$\lim_{r\to\infty}[n(r,\ \theta-\varepsilon,\ \theta+\varepsilon,\ f=a)+n(r,\ \theta-\varepsilon,\ \theta+\varepsilon,\ f^{(l)}=b)]=\infty \tag{1}$$

holds for all $(a,\ b)\in C\times[C-0]$, where

$$n(r,\ \theta-\varepsilon,\ \theta+\varepsilon,\ g=\beta)$$

is the number of roots of $g-\beta=0$ in the region

$$[|z|<r]\cap[|\arg z-\theta|<\varepsilon].$$

Yang, Lo [1] proved that for given meromorphic function f there is a ray $\arg z=\theta$ which satisfies (1) provided that

$$\limsup_{r\to\infty}\frac{T(r,\ f)}{(\log r)^3}=\infty . \tag{2}$$

A problem posed in [3] asks whether (2) could be replaced by the usual existing condition of classical Julia directions

$$\limsup_{r\to\infty}\frac{T(r,\ f)}{(\log r)^2}=\infty . \tag{3}$$

In this paper we prove that there is a ray $\arg z=\theta$ satisfying (1) provided that (3) holds.

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