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ON RESOLUTION COMPLEXITY OF PLANE CURVES

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Introduction

The embedded resolution of a plane curve is a known process which was already known by Max Noether in [N]. One may find an elementary account of this process in the book of E. Brieskorn and H. Knörrer [B-K].

When the plane curve is locally irreducible, it is easily observed that the case of one Puiseux pair is solved by only one good toroidal blowing-up. More generally with one good toroidal blowing-up one may eliminate the first Puiseux pair. Therefore after g good toroidal blowing-ups, one can solve a curve singularity with one branch and g Puiseux pairs.

In this paper we show that this phenomenon is general. Namely we prove that the minimal number of required toroidal blowing-ups to resolve the curve singularity is a topological invariant of the singularity that we have called the complexity of the resolution (see Theorem (3.12)).

In fact the complexity of a plane curve singularity is expected to behave like a depth. If we expect that a non-degenerate hypersurface singularity has resolution complexity one, it is reasonable to prove that a general plane section of that hypersurface is a curve which singularity has complexity at most equal to the dimension of the hypersurface. This result was actually proved by the second author in [O4].

It remains to understand what is the resolution complexity of a hypersurface singularity. We hope that this paper will draw the interest of the specialists on this subject.

1. Choice of good coordinates

Let $f(x, y)$ be a complex analytic function of two variables defined on an open neighborhood U of the origin O of $\boldsymbol{C}^2$ and suppose that $f(O)=0$. Let

$$f(x, y)=\sum c_{\alpha,\beta}x^{\alpha}y^{\beta}$$

be the Taylor expansion of f at the origin $O=(0, 0)$. *The Newton polygon*

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