S. IZUMIYA AND Y KUROKAWA
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ON SYSTEMS OF CLAIRAUT TYPE

BY SHYUICHI IZUMIYA AND YASUHIRO KUROKAWA

0. Introduction

About 260 years ago Alex Claude Clairaut [1] studied the following equation which is called the Clairaut equation now : $y = x \cdot \frac{dy}{dx} + f(\frac{dy}{dx})$. It is usually taught in the first or second year course of calculus in the university and treated as one of the typical examples of non-linear equations that are easily solved. Moreover it has a quite beautiful geometric structure as follows : There exists a "general solution" that consists of lines ; $y = t \cdot x + f(t)$, where t is a parameter and the singular solution is the envelope of such a family (Fig. 1).

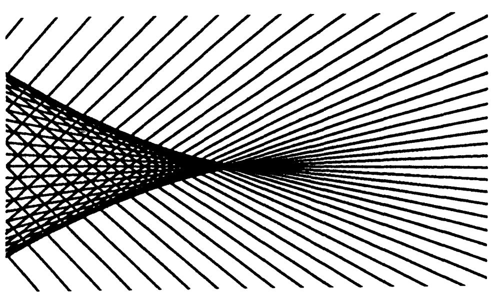

Fig. 1

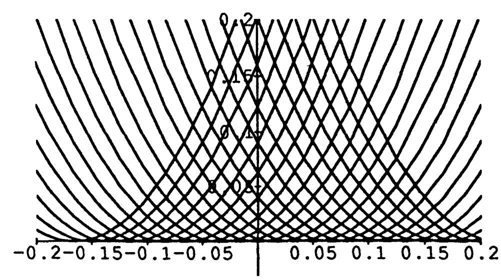

Fig. 2

In this article we consider equations with the same geometric structure as the Clairaut equation. Here we give another example as follows : $y - (\frac{dy}{dx})^2 = 0$. We can easily solve this equation: the "general solution" is given by $y = \frac{1}{4}(x+t)^2$, where t is a parameter. Here, the "singular solution" is given by $y = 0$ that is the envelope of the family of graphs of the "general solution". The "general solution" of this equation does not consist of lines. However, the "singular solution" is the envelope of the graphs of the "general solution" like as the Clairaut equation (Fig. 2).

We will refer such an equation as a Clairaut-type equation. Our purpose in this note is to give a characterization of Clairau-type equations and to classify these equation under a "nice" equaivalence relation.

In this note we will sketch our discussions without proofs. The detailed paper will be published elswhere soon.

All maps considered here are differentiable of class C^∞, unless stated otherwise.

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