NGUYEN SUM
KODAI MATH. J.
17 (1994), 585—595

STEENROD OPERATIONS ON THE MODULAR INVARIANTS

BY NGUYEN SUM

Introduction

Fix an odd prime p. Let A_{p^n} be the alternating group on p^n letters. Denote by $\Sigma_{p^n,p}$ a Sylow p-subgroup of A_{p^n} and E^n an elementary abelian p-group of rank n. Then we have the restriction homomorphisms

$$\mathrm{Res}(E^n, \Sigma_{p^n,p}) : H^*(B\Sigma_{p^n,p}) \longrightarrow H^*(BE^n),$$
$$\mathrm{Res}(E^n, A_{p^n}) : H^*(BA_{p^n}) \longrightarrow H^*(BE^n),$$

induced by the regular permutation representation $E^n \subset \Sigma_{p^n,p} \subset A_{p^n}$ of E^n (see Mùi [4]). Here and throughout the paper, we assume that the coefficients are taken in the prime field $\mathbf{Z}/p$. Using modular invariant theory of linear groups, Mùi proved in [3], [4] that

$$\mathrm{ImRes}(E^n, \Sigma_{p^n,p}) = E(U_1, \ldots, U_n) \otimes P(V_1, \ldots, V_n),$$
$$\mathrm{ImRes}(E^n, A_{p^n}) = E(\tilde{M}_{n,0}, \ldots, \tilde{M}_{n,n-1}) \otimes P(\tilde{L}_n, Q_{n,1}, \ldots, Q_{n,n-1}).$$

Here and in what follows, $E(.,\ldots,.)$ and $P(.,\ldots,.)$ are the exterior and polynomial algebras over $\mathbf{Z}/p$ generated by the variables indicated. $\tilde{L}_n, Q_{n,s}$ are the Dickson invariants of dimensions p^n, $2(p^n - p^s)$, and $\tilde{M}_{n,s}$, U_k, V_k are the Mùi invariants of dimensions $p^n - 2p^s$, p^{k-1}, $2p^{k-1}$ respectively (see §1).

Let A be the mod p Steenrod algebra and let τ_s, ξ_i be the Milnor elements of dimensions $2p^s - 1$, $2p^i - 2$ respectively in the dual algebra A_* of A. In [7], Milnor showed that, as an algebra

$$A_* = E(\tau_0, \tau_1, \ldots) \otimes P(\xi_1, \xi_2, \ldots).$$

Then A_* has a basis consisting of all monomials $\tau_S \xi^R = \tau_{s_1} \ldots \tau_{s_k} \xi_1^{r_1} \ldots \xi_m^{r_m}$, with $S = (s_1, \ldots, s_k)$, $0 \le s_1 < \ldots < s_k$, $R = (r_1, \ldots, r_m)$, $r_i \ge 0$. Let $St^{S,R} \in A$ denote the dual of $\tau_S \xi^R$ with respect to that basis. Then A has a basis consisting all operations $St^{S,R}$. For $S = \emptyset$, $R = (r)$, $St^{\emptyset,(r)}$ is nothing but the Steenrod operation P^r.

Since $H^*(BG)$, $G = E^n$, $\Sigma_{p^n,p}$ or A_{p^n}, is an A-module (see [13 ; Chap. VI]) and the restriction homomorphisms are A-linear, their images are A-submodules of $H^*(BE^n)$.

The purpose of the paper is to study the module structures of $\mathrm{ImRes}(E^n, \Sigma_{p^n,p})$ and $\mathrm{ImRes}(E^n, A_{p^n})$ over the Steenrod algebra A. More precisely, we prove a duality relation between $St^{S,R}(\tilde{M}_{n,s}^{\delta} Q_{n,s}^{1-\delta})$ and $St^{S',R'}(U_{k+1}^{\delta} V_{k+1}^{1-\delta})$ for $\delta = 0, 1$, $\ell(R) = k$ and $\ell(R') = n$.

Received May, 14, 1993.