Y MATSUMOTO
KODAI MATH. J.
17 (1994), 560—570

ON THE TOPOLOGICAL STRUCTURE OF THE FERMAT SURFACE OF DEGREE 5

BY YUKIO MATSUMOTO

Abstract

We will give the monodromy representation of a certain fibration of the Fermat surface of degree 5 explicitly in terms of Dehn twists about concrete curves. This paper is a sequel to Ahara's work [1]

1. Introduction

Let V_5 be the complex projective hypersurface in CP_3 defined by the equation of degree 5

$$z_0^5 - z_1^5 = z_2^5 - z_3^5$$

where z_0,z_1,z_2,z_3 are the homogenuous coordinates. From topological viewpoint, this surface is a simply-connected 4-manifold. The Euler characteristic and the signature of V_5 are equal to 55 and -35, respectively (see [5]). By Freedman [4] V_5 is homeomorphic to $9CP_2\#44\overline{CP}_2$, but by Donaldson [3] it is not diffeomorphic to this connected sum.

Our motive is to understand the topological structure of V_5 through a holomorphic fibration over the Riemann sphere $CP_1 = C \cup \{\infty\}$.

The fibration $f : V_5 \to CP_1$ is defined as follows:

$$f : [z_0, z_1, z_2, z_3] \mapsto \begin{cases} z_2^4/z_0^4, & \text{if } z_0 = z_1 \text{ and } z_2 = z_3 \\ (z_0 - z_1)/(z_2 - z_3), & \text{otherwise.} \end{cases}$$

A general fiber of this fibration is a Riemann surface of genus 3. In unpublished notes (1990), the author determined the positions and the topological types of all singular fibers in $f : V_5 \to CP_1$; let F_σ denote the fiber over a point $\sigma \in CP_1$. Then F_σ is a singular fiber if and only if σ belongs to the following set SF consisting of 17 points:

$$SF = \{\sigma \mid \sigma^5 = -1/4, 1, or - 4\} \cup \{0, \infty\}.$$

If $\sigma = 0$ or ∞ , F_σ is a union of 4 complex lines meeting in a point. If σ is a 5-th root of 1, F_σ is a union of two complex lines and a conic, meeting in 5 points transversely. If σ is a 5-th root of $-1/4$ or of-4, F_σ is an irreducible stable curve of virtual genus 3

1991 *Mathematics Subject Classfication.* Primary 57M50, Secondary 32J15
This paper is dedicated to the memory of Professor Yukiyosi Kawada
Received October 30, 1993.