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KODAI MATH. J.
17 (1994), 549—559

SURFACES IN 3-MANIFOLDS: GROUP ACTIONS ON SURFACE BUNDLES

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Abstract

We discuss briefly some conjectures concerning surfaces in 3-manifolds, and describe two results on finite group actions on surface bundles over the circle.

1. Manifolds of dimension 1 and 2

The classification of closed orientable surfaces can be achieved by studying embedded circles on surfaces. Analogously, approaches to the classification of closed orientable 3 dimensional manifolds involve studying embedded circles and surfaces. We review these approaches, often illustrating more general results by their statements in the closed, orientable context: unless otherwise clear, manifolds considered will be closed (compact with empty boundary), connected, and orientable. Manifolds and maps are smooth. References are necessarily incomplete.

Any closed connected 1-manifold is diffeomorphic to the circle S^1. In dimension 2, either genus or Euler characteristic determine a closed orientable surface, as does the fundamental group. Any closed, orientable surface S can be decomposed as a connect-sum $S \cong \#_k T^2 = \#_k S^1 \times S^1$ of k tori, for a unique $k \geqq 0$. (We denote by $M \# N$ the *connect sum* of two n-manifolds M and N, obtained by removing an open ball from each, and identifying the two resulting boundary spheres.)

There are 3 classes of surface, according to whether the Euler characteristic is positive, zero, or negative. These correspond to the sphere S^2, the torus $S^1 \times S^1$, and surfaces of higher genus; these classes correspond to the 3 types of homogeneous 2-dimensional geometry. Representatives admit metrics of constant curvature $+1$, 0 and -1 respectively. Moreover, fundamental groups are respectively finite (in fact, trivial), infinite abelian (polynomial growth), and infinite non-abelian (exponential growth). Surface types play different but important roles in the study of 3-dimensional manifolds. As motivation, we consider common approaches to classifying a surface S. These usually involve cutting S along π_1-*injective* circles: these are circles representing non-trivial homotopy classes. Very importantly, if a surface contains an *immersed* π_1-injective circle, it contains an *embedded* π_1-injective circle.

One approach is to seek an embedded circle γ which is non-trivial homologically on S, and which is thus *non-separating*. Another embedded circle γ' meeting γ transversely at one point can easily be found. Each of these circles has an annular neighbourhood on the surface, chosen so that their union is a torus with a disc removed. The boundary of

1991 subject classification (Amer. Math. Soc.): 57N10, 57M35, 57M50.

Received October 28, 1993.