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THE RECOGNITION PROBLEM FOR TOPOLOGICAL MANIFOLDS: A SURVEY

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Abstract

This is a survey of recent work on the problem of recognizing topological manifolds among topological spaces, including the results of J.L. Bryant, S.C. Ferry, W. Mio and S. Weinberger in higher dimensions, and M.V. Brahm, R.J. Daverman and D. Repovš in dimension three.

1. Introduction

The definition of a *topological n-manifold* M ($n \in \mathbf{N}$) is quite simple to state – besides the separability and metrizability we require (and this is the key *geometric* property of manifolds) that every point $x \in M$ should possess a neighbourhood $U \subset M$ which is *homeomorphic* to $\mathbf{R}^n$. (We shall only consider *closed* topological manifolds M, i.e. M is connected, compact and $\partial M = \emptyset$.) However, in practice, it is precisely the *verification* of the existence (or nonexistence) of such homeomorphism $h : U \to \mathbf{R}^n$, which is the biggest problem. So a natural question arises: *Is it possible to find a characterization of topological manifolds which does not mention homeomorphisms, which is reasonably simple to state but which is also not too difficult to verify?*

This is the so-called *Recognition problem for topological manifolds*, one of the most important problems of *geometric topology*, i.e. of its branch called *Bing* (or *Texas*) *topology*. In the present paper we plan to survey the most recent work on this problem, including the results due to J.L. Bryant, S.C. Ferry, W. Mio and S. Weinberger [10] in higher dimensions and M.V. Brahm [7], R.J. Daverman and D. Repovš [18] [19] in dimension three. For an account of the work done earlier see the surveys of J.W. Cannon [13] and D. Repovš [34] [35] (the survey [35] also contains an extensive bibliography on this subject). For the closely related topics — the topology of cell-like maps — the reader may wish to consult the latest survey by W.J.R. Mitchell and D. Repovš [26].

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