S. KOJIMA
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GEOMETRY OF HYPERBOLIC 3-MANIFOLDS WITH BOUNDARY

BY SADAYOSHI KOJIMA

§1. Hyperbolic Manifolds with Boundary

A hyperbolic manifold will be a complete Riemannian manifold of constant sectional curvature -1. We call a simply connected hyperbolic manifold the hyperbolic space in particular. One standard model of the hyperbolic space is a ball $\mathbf{P} = \{(x_1, \cdots, x_n) \in \mathbf{R}^n \mid x_1^2 + \cdots + x_n^2 < 1\}$ equipped with the Poincaré metric

$$\frac{4(dx_1^2 + \cdots + dx_n^2)}{(1 - (x_1^2 + \cdots + x_n^2))^2}.$$

A hyperbolic manifold is, in other words, a complete Riemannian manifold locally modelled on the Poincaré ball.

Particular interests have been paid to the case in dimension 3 by many mathematicians of various fields since late 70's, probably because really significant contributions by Thurston, Gromov, Sullivan and many others justify that their mathematical structures behind appearance is undoubtedly rich. Thurston's lecture note [21] has been serving as a pioneering bible, and many other related articles such as [3,5] have appeared in these days.

In this note, we focus on more restrictive class, a hyperbolic 3-manifold of finite volume with non-empty totally geodesic boundary, and would like to report our naive studies on their geometry. Throughout the sequel, we adopt a notation N to indicate such a manifold without referring conditions. N will hence always mean a hyperbolic 3-manifold of finite volume with totally geodesic boundary.

Even with these additional requirements, N is still very attractive and enjoys many nice properties. At first, they are almost relatives to complete hyperbolic 3-manifolds and have many properties in common with them. They have also their own features. For example, they can be rather easily constructed, they have visible sides interplaying in geometry and topology, they are related to well established Teichmüller theory, and so on.

To advertise a flavor of naive and pleasant study of N, let us look at the universal covers. The universal cover $\widetilde{N}$ of N is developed in $\mathbf{P}$ as a convex subset with geodesic boundary. If we see $\widetilde{N}$ from outside the Poincaré ball, each component of the boundary appears much like a round hole on a golf ball surface. Its edge defines a circle on the sphere at infinity $\mathbf{P}_\infty = \{x_1^2 + x_2^2 + x_3^2 = 1\}$ by taking closure in the euclidean topology. $\partial\widetilde{N}$ has infinitely many components and the set of all circles appeared by closure forms a circle packing $\mathcal{C}_N$ on $\mathbf{P}_\infty$. $\mathcal{C}_N$ has a completeness property due to finiteness of $\operatorname{vol} N$,

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