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ON THE SPATIAL GRAPH

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In this article we will explain about spatial graph. Spatial graph is a spatial presentation of a graph in the 3-dimensional Euclidean space $\mathbf{R}^3$ or the 3-sphere S^3. That is, for a graph G we take an embedding $f : G \to \mathbf{R}^3$, then the image $\widetilde{G} := f(G)$ is called a spatial graph of G. So the spatial graph is a generalization of knot and link. For example the figure 0 (a), (b) are spatial graphs of a complete graph with 4 vertices.

(a) (b)

Fig. 0.

Spatial graph theory has an application for molecular biology or stereochemistry to distinguish topological isomer. In this paper we will assume all homeomorphisms and embeddings piecewise linear or edgewise differentiable unless otherwise is stated. To distinguish spatial graphs there are nine equivalence relations among them;

(1) ambient isotopic (2) homeomorphic as a pair
(3) cobordant (4) isotopic
(5) I-equivalent (6) graph homotopic
(7) weakly graph homotopic (8) graph homologous
(9) $\mathbf{Z}_2$ graph homologous.

Those definitions are as follows. Let $f, g : G \to \mathbf{R}^3$ be spatial presentations of G and $I = [0,1]$ a unit interval.

Then f and g are

(1) ambient isotopic if there is a level preserving locally flat embedding $\Phi : G \times I \to \mathbf{R}^3 \times I$ between f and g that is, $\Phi(G,0) = f(G)$, $\Phi(G,1) = g(G)$,
(2) homeomorphic as a pair if there is a homeomorphism $\Phi : (\mathbf{R}^3, f(G)) \to (\mathbf{R}^3, g(G))$,
(3) cobordant if there is a locally flat embedding $\Phi : G \times I \to \mathbf{R}^3 \times I$ between f and g,
(4) isotopic if there is a level preserving embedding $\Phi : G \times I \to \mathbf{R}^3 \times I$ between f and g,
(5) I-equivalent if there is an embedding $\Phi : G \times I \to \mathbf{R}^3 \times I$ between f and g,
(6) graph homotopic if g is obtained from f by a series of self-crossing changes and

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