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ON LENGTH-MINIMIZING STEINER NETWORKS

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Introduction

Let M be a set of points in R^n. The Steiner problem can be stated as follows: to find a network of least length in the class of networks with fixed ends M. There are two approaches to this problem. In the first case, the search for an absolutely minimal network is carried out in the class of networks, whose vertices all belong to M. This approach is developed in the papers by Du, Hwang and others (see, for example, [5], [6]). In the second case, the set of vertices of the networks may be larger than the set M. The vertices do not belong to M are called Steiner points and the points in M are called boundary points. In this paper we study globally minimal network in the second case. By using calibration systems we prove that each locally minimal network is also globally minimal in the class of networks with the same topological type. The method of calibrations was developed in works of Federer, Dao Trong Thi, Lawson, Harvey, Morgan and others (see, for example, [1]-[3]). Calibration systems were used first in [4].

1. Steiner networks

DEFINITION 1.1. A Steiner network in R^n is any connected complex of one-dimensional simplexes, whose vertices have degree at most three. A Steiner network without vertices of degree two is said to be nondegenerate Steiner network.

Henceforth we shall study only acyclic nondegennerate Steiner networks with boundary points coinsiding with the vertices of degree one. Such networks are called simply Steiner networks.

LEMMA 1.2. *Let a Steiner network N has k boundary points. Then N has $(k-2)$ Steiner points and $(2k-3)$ sides.*

Proof. Assume that the network N has m Steiner points and c sides. By calculating we have $k = 3m - 2(m-1)$. Hence, we obtain $m = k-2$ and $c = 2k-3$. The lemma is proved.

DEFINITION 1.3. Two boundary points of a Steiner network are called adjacent boundary points if they are ends of two adjacent sides (i.e. sides, which have a common end).

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