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KODAI MATH. J.
17 (1994), 463—475

GROUP ACTIONS AND DEFORMATIONS FOR HARMONIC MAPS INTO SYMMETRIC SPACES

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Let M and N be Riemannian manifolds. The *energy* of a smooth map $\varphi : M \longrightarrow N$ is defined as

$$E(\varphi) = \frac{1}{2} \int_M \|d\varphi\|^2 dv.$$

A smooth map φ is called *harmonic* if the first variation of the energy vanishes for every smooth variation of φ with compact support. In the case $\dim M = 2$, since the energy is invariant under conformal deformations of the Riemannian metric on M, it is natural to consider a Riemann surface Σ rather than a Riemannian manifold M as the domain manifold.

This article is concerned with two related areas of harmonic map theory ; group actions and deformations for harmonic maps of Riemann surfaces Σ into symmetric spaces N.

The action of a certain infinite dimensional Lie group and algebra on harmonic maps from a simply connected Riemann surface, especially a Riemann sphere, into a compact Lie group or a symmetric space have been investigated by Uhlenbeck [Uh], Zakharov-Shabat-Mikhailov [ZM,ZS] others. In a joint paper with M.A. Guest ([GO1]), we have shown how the action of a infinite dimensional Lie group can be interpreted in terms of the Grassmannian model in Loop Group Theory (cf. [PS],[Se]) and investigated its geometric nature, and as an important application we discussed deformations of harmonic maps from the viewpoint of Morse-Bott theory over twistor spaces. Using this idea, we have given results on the connectedness of spaces of harmonic 2-spheres in the standard sphere S^n, the real projective space $\mathbf{R}P^n$ (see also [Ko]) and the complex projective space $\mathbf{C}P^n$, the unitary group $U(n)$, and in [FGKO] we have determined the fundamental group of the space of harmonic 2-spheres in S^n.

In Section 1 and 2 we shall review the construction of extended solutions of harmonic maps into Lie groups and the natural action of the complex loop group on harmonic maps into Lie groups. In Section 3 we shall discuss group actions on harmonic maps into symmetric spaces of inner type and in Section 4 we shall mention further results on the connectedness of certain spaces of harmonic 2-spheres in symmetric spaces. These are joint works with M.A. Guest and M.Mukai in progress.

1. Extended solutions for harmonic maps

Let us begin with the definition of the notion of extended solutions of harmonic maps into Lie groups. Let G be a compact connected Lie group and $\mathfrak{g}$ be its Lie algebra. We

Received October 15, 1993.