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# VOLUME MINIMIZING SUBMANIFOLDS IN COMPACT SYMMETRIC SPACES

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In this note we consider two methods in order to investigate volume minimizing submanifolds in compact symmetric spaces. The first is calibration ([4]) and the second is integral geometry. We can show that certain submanifolds are volume minimizing in their real homology classes using calibrations. A calibration is a closed differential form on a Riemannian manifold which satisfies a certain inequality. A definition of calibrations will be given in Section 1. On the othor hand we can prove that certain submanifolds are volume minimizing in its homotopy classes using integral geometry. We shall use a generalized Poincaré's formula in Riemannian homogeneous spaces given by Howard [7].

## 1. Calibrations

Let $M$ be a Riemannian manifold with a closed $p$-form $\phi$ on $M$ which satisfies the following inequality:

$$\phi|_\xi \leq \mathrm{vol}_\xi$$

for any oriented tangent $p$-plane $\xi$ on $M$. Such a form $\phi$ is called a *calibration*. Then any compact oriented $p$-dimensional submanifold $N$ in $M$ with the property:

$$\phi|_N = \mathrm{vol}_N$$

is volume minimizing in $M$, that is,

$$\mathrm{vol}(N) \leq \mathrm{vol}(N')$$

for any compact oriented $p$-dimensional submanifold $N'$ such that $[N] = [N']$ in the homology group $H_p(M;\mathbf{R})$. We say that $\phi$ *calibrates* $N$. Using Stokes' theorem we get

$$\mathrm{vol}(N) = \int_N \mathrm{vol}_N = \int_N \phi = \int_{N'} \phi \leq \int_{N'} \mathrm{vol}_{N'} = \mathrm{vol}(N').$$

The equality holds if and only if $\phi|_{N'} = \mathrm{vol}_{N'}$.

The fundamental 2-form of a Kählar manifold is one of important examples of clibrations. It satisfies Wirtnger's inequality, which is stated as follows. Let $M$ be a Kähler manifold with fundamental 2-form $\omega$. Then

$$\frac{1}{k!}\omega^k|_\xi \leq \mathrm{vol}_\xi$$

for $1 \leq k \leq \dim_C M$ and any oriented tangent $2k$-plane $\xi$ on $M$. Therefore $\frac{1}{k!}\omega^k$ is a calibration on $M$. The equality holds if any only if $\xi$ is a complex plane with a canonical

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