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ON MINIMAL SURFACES WITH THE RICCI CONDITION IN SPACE FORMS

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0. Introduction

A 2-dimensional Riemannian metric ds^2 is said to satisfy the Ricci condition with respect to c if its Gaussian curvature K satisfies $K < c$ and the new metric $d\hat{s}^2 = \sqrt{c-K}ds^2$ is flat.

Let $X^N(c)$ denote the N-dimensional simply connected space form of constant curvature c, and in particular, let $\mathbf{R}^N = X^N(0)$. The induced metric ds^2 on a minimal surface in $X^3(c)$ satisfies the Ricci condition with respect to c except at points where the Gaussian curvature $= c$. Conversely, assume that a Riemannian metric ds^2 on a 2-dimensional simply connected manifold M satisfies the Ricci condition with respect to c. Then there exists a smooth 2π-periodic family of isometric minimal immersions $f_\theta : (M, ds^2) \to X^3(c)$; $\theta \in \mathbf{R}$, which is called the associated family. Moreover, up to congruences, the maps f_θ; $0 \leqq \theta < \pi$ represent all local isometric minimal immersions of (M, ds^2) into $X^3(c)$ (see [5]). So, the Ricci condition with respect to c is an intrinsic characterization of minimal surfaces in $X^3(c)$.

Here we consider the following problem, which may be seen as a kind of rigidity problem.

PROBLEM. *Classify those minimal surfaces in $X^N(c)$ whose induced metrics satisfy the Ricci condition with respect to c, or equivalently, classify those minimal surfaces in $X^N(c)$ which are locally isometric to minimal surfaces in $X^3(c)$.*

A submanifold in $X^N(c)$ is said to lie fully in $X^N(c)$ if it does not lie in a totally geodesic submanifold of $X^N(c)$. Let $S(N, c)$ denote the set of all Riemannian structures of minimal surfaces lying fully in $X^N(c)$. Then the problem is to determine the intersection of $S(3, c)$ and $S(N, c)$.

1. Examples

In this section, we give examples of minimal surfaces in $X^N(c)$ which do not lie in a totally geodesic $X^3(c)$ and whose induced metrics satisfy the Ricci condition with respect to c. The following three types of examples are known.

Example 1 ([6]). Let $f_\theta : (M, ds^2) \to \mathbf{R}^3$; $\theta \in \mathbf{R}$ be the associated family of isometric minimal immersions of a 2-dimensional Riemannian manifold (M, ds^2) into

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