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THE REAL PART OF DECOMPOSITION OF A POLYNOMIAL AND ITS DETERMINACY

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1. Introduction

Let $f(x,y)$, $g(x,y)$: $(\mathbb{R}^2,0) \to (\mathbb{R},0)$ be two C^∞ function-germs. Germs f and g are called to be r-jet equivalent if at $(0,0)$, their derivatives of degree not greater than r are identical. Denote this fact by $j^r(f) = j^r(g)$. Germ f is called to be C^0-r-determined if for each germ g with $j^r(f) = j^r(g)$, there exists a germ of homeomorphism $h : (\mathbb{R}^2,0) \to (\mathbb{R}^2,0)$ such that $f \circ h = g$. f is called to be C^0-finitely-determined if it is C^0-r-determined for some r. The degree of C^0-determinacy of f is the least number such that f is C^0-r-determined.

Germs f and g are called to be V-equivalent if germs $f^{-1}(0)$ and $g^{-1}(0)$ are homeomorphic.

Let $P_0(n,k;\mathbb{R})$ denote the set of topological equivalence classes of germs of real polynomials in n variables of degree $\leq k$, and $P_0(n,\mathbb{R})$ the set of those classes for all k. T. Fukuda [1] proved the Thom's conjecture: $P_0(n,k;\mathbb{R})$ is a finite set. How about $P_0(n;\mathbb{R})$? It is easy to see that $P_0(1;\mathbb{R})$ contains only three elements. For example, the germs $y = x^2$ and $y = x^4$ are C^0-equivalent (V.I. Arnol'd etc. [2], p. 12). In general, $y = x^{2m}$ and $y = x^{2n}$ belong to be the same class, and $y = x^{2m+1}$ and $y = x^{2n+1}$ belong to be the another class.

2. Homogeneous case

Let $P(x,y)$ be a germ of a real homogeneous polynomial of degree k. Then

$$P(x,y) = a(x-b_1y)\cdots(x-b_sy)(x-c_1y)\cdots(x-c_my)$$

where $a, b_i \in \mathbb{R}, a \neq 0, c_j \in \mathbb{C}$. We have the following.

THEOREM 1. *$P(x,y)$ is C^0-finitely determined if and only if $b_i \neq b_j$ for $i \neq j$. In this case, the degree of C^0-determinacy of P is k.*

THEOREM 2. *Homogeneous polynomial-germs $P(x,y)$ and $Q(x,y)$ are V-equivalent if and only if they have the same number of real factors (do not account the repeated number, if $b_i = b_j$ for some i,j).*

Remark. The degrees of P and Q may be unequal when they are V-equivalent.

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