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KODAI MATH. J.
17 (1994), 432—437

MODIFIED NASH TRIVIALITY OF A FAMILY OF ZERO-SETS OF WEIGHTED HOMOGENEOUS POLYNOMIAL MAPPINGS

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§0. Introduction.

Let $\alpha = (\alpha_1, \cdots, \alpha_n)$ be an n-tuple of positive integers. Assume that the greatest common divisor of α_j's is 1. Let $\mathbf{N}$ denote the set of positive integers, and let $\mathbf{R}$ denote the set of real numbers. Let $f : (\mathbf{R}^n, 0) \to (\mathbf{R}, 0)$ be a polynomial function defined by

$$f(x) = \sum_\beta A_\beta x_1^{\beta_1} \cdots x_n^{\beta_n} \quad (A_\beta \neq 0,\ \beta_1, \cdots, \beta_n \in \mathbf{N} \cup \{0\}).$$

We say that f is *weighted homogeneous of type* $(\alpha_1, \cdots, \alpha_n; L)$ $(\alpha_1, \cdots, \alpha_n, L \in \mathbf{N})$, if

$$\alpha_1\beta_1 + \cdots + \alpha_n\beta_n = L \quad \text{for any } \beta = (\beta_1, \cdots, \beta_n).$$

Let J be an open interval, and $t_0 \in J$. Let $f_t : (\mathbf{R}^n, 0) \to (\mathbf{R}^p, 0)$ be a polynomial mapping where each $f_{t,i}$ is weighted homogeneous of type $(\alpha_1, \cdots, \alpha_n; L_i)$ $(1 \leqq i \leqq p)$ for $t \in J$. We define a mapping $F : (\mathbf{R}^n \times J, \{0\} \times J) \to (\mathbf{R}^p, 0)$ by $F(x,t) = f_t(x)$. Assume that F is a polynomial mapping (or of class C^2). It is well-known that the following fact holds under these assumptions:

FACT. *If $f_t^{-1}(0) \cap \sum f_t = \{0\}$ for any $t \in J$ (where $\sum f_t$ denotes the singular points set of f_t), then $(\mathbf{R}^n \times J, F^{-1}(0))$ is topologically trivial i.e. there exists a t-level preserving homeomorphism $\sigma : (\mathbf{R}^n \times J, \{0\} \times J) \to (\mathbf{R}^n \times J, \{0\} \times J)$ such that*

$$\sigma((R^n \times J, F^{-1}(0))) = (\mathbf{R}^n \times J, f_{t_0}^{-1}(0) \times J).$$

Remark 1. Results generalizing this fact have been obtained in [2], [5]. But it seems that the fact itself was recognized by many mathematicians a good while ago.

Since we consider the weighted homogeneous case with an isolated singularity, it seems natural that stronger triviality than topological one holds. In fact, such triviality called "modified Nash triviality" holds under the above assumptions (see Theorem in §2). On the other hand, we have introduced the notion of "strong C^0 triviality" for a family of analytic functions in [6]. Roughly speaking, strong C^0 equivalence is a C^0 equivalence which preserves the tangency of analytic arcs at $0 \in \mathbf{R}^n$. In §4, we discuss the relation between modified Nash triviality and strong C^0 triviality for a family of zero-sets of weighted homogeneous polynomials.

The author would like to thank Professor M. Shiota for helpful suggestions concerning Proposition 2 in §3 (semi-algebraic triviality theorem).

Received April 30, 1993.