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ON THE FUNDAMENTAL GROUP OF THE COMPLEMENT OF ARRANGEMENTS

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Let V be a vector space of finite dimension. An arrangement of hyperplanes in V is a finite collection $\mathcal{A}$ of hyperplanes of V. An arrangement $\mathcal{A}$ will be said to be real (resp. complex) if V is a real (resp. complex) vector space. The complexification of a hyperplanes H of $\mathbf{R}^n$ is the hyperplane $H_{\mathbf{C}}$ of $\mathbf{C}^n$ having the same equation as H. Given an arrangement $\mathcal{A}$ in $\mathbf{R}^n$, we have its complexification $\mathcal{A}_{\mathbf{C}}$ to be the complex arrangement $\{H_{\mathbf{C}}; H \in \mathcal{A}\}$ in $\mathbf{C}^n$.

Given an arrangement $\mathcal{A}$, we are interested in finding a presentation for the fundamental group $\pi_1(M)$ of the complement

$$M = V - \bigcup_{H \in \mathcal{A}} H$$

in case $\mathcal{A}$ is a complex arrangement, and $\pi_1(M_{\mathbf{C}})$ of the complement of $\mathcal{A}_{\mathbf{C}}$ in case $\mathcal{A}$ is a real arrangement. In [2] we have suggested a geometrical method to compute the fundamental group of a manifold equipped with a suitable cellular decomposition. Also, given a real arrangement $\mathcal{A}$ in $\mathbf{R}^n$, we have introduced a certain cellular decomposition $\mathcal{C}(\mathbf{C}^n, \mathcal{A})$ of $\mathbf{C}^n$, induced from the arrangement $\mathcal{A}$. In this note, we will apply our method to this decomposition to find a presentation for $\pi_1(M_{\mathbf{C}})$ of any real arrangement $\mathcal{A}$. Such a presentation has been given by M. Salvetti in [4] using his complex. After reducing the problem to the case of dimension 2, W. Arvola has suggested an algorithm to find a presentation for the complement of a complex arrangement. In a sequent paper [3] we will also treat the case when $\mathcal{A}$ is a complex arrangement.

We first recall of our method suggested in [2]. Let $\mathcal{M}$ be a connected topological-manifold of dimension n with a locally finite CW-semicomplex structure $\mathcal{C}_{\mathcal{M}}$ such that $\mathcal{M}$ is 1-codimensionally regular (see [2] for the notion of CW-semicomplex and 1-codimensional regularity). Each $(n-1)$-cell σ of $\mathcal{M}$ is a face of exactly two n-cells, say c and c'. Then we have two n-intervals $[c, \sigma, c']$ and $[c', \sigma, c]$. We specify one of them by $[\sigma]$ and the other by $[\sigma]^{-1}$. A n-path γ on $\mathcal{M}$ is a join of a finite number of n-intervals

$$\gamma = [\sigma_1]^{\epsilon_1} \vee [\sigma_2]^{\epsilon_2} \vee \cdots \vee [\sigma_k]^{\epsilon_k},$$

where $\epsilon_i = \pm 1$, σ_i are $(n-1)$-cells of $\mathcal{M}$, $1 \leq i \leq k$. If $[\sigma_1]^{\epsilon_1} = [c, \sigma_1, c_1]$ and $[\sigma_k]^{\epsilon_k} = [c_k, \sigma_k, c']$, for some n-cells c, c_1, c_k and c' we say that γ is a n-path from c to c'. Among n-paths on the manifold $\mathcal{M}$ we have defined in [2] a certain equivalence relation.

Let $\mathcal{M}$ be given a base point $*$ belonging to a certain n-cell c_0. Then, the equivalence classes of closed n-paths at $*$ form a group denoted by $\pi_1(\mathcal{C}_{\mathcal{M}}, *)$. In [2] we have proved the isomorphism $\pi_1(\mathcal{C}_{\mathcal{M}}, *) \cong \pi_1(\mathcal{M}, *)$. So, in order to compute $\pi_1(\mathcal{M}, *)$, it suffices to compute $\pi_1(\mathcal{C}_{\mathcal{M}}, *)$. And the latter can be determined by means of the decomposition $\mathcal{C}_{\mathcal{M}}$ as below

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