S. ISHII
KODAI MATH. J.
17 (1994), 420—427

A FANO 3-FOLD WITH NON-RATIONAL SINGULARITIES AND A TWO DIMENSIONAL BASIS

BY SHIHOKO ISHII

Introduction

In this paper, the author gives a summary of the paper [I2] and pictures of the Fano 3-folds which appear in it. Here a Fano 3-fold means a normal projective variety of dimension three over C whose anticanonical sheaf is ample and invertible. During the past fifteen years, there has been big progress in the investigation of a non-singular Fano 3-fold owing to Iskovskih, Mori, Mukai and Shokurov. And it is still developing. On the other hand, in singular Fano 3-folds, progress seems to have started recently. Here we study the structure of a Fano 3-fold with non-rational singularities.

Let Σ be the locus of non-rational singular points of a Fano 3-fold X. As X is normal, $\dim\Sigma \leqq 1$. If $\dim\Sigma = 0$, then X is isomorphic to a projective cone over a normal K3-surface or an Abelian surface (Theorem 1A, 1B). The proof of this theorem also works in the case that Σ contains an isolated point. So what we should study next is the case that Σ has pure dimension one. Such a Fano 3-fold is classified in three families according to the maximal basis-dimension of its $\mathbf{Q}$-factorial terminal modification (Theorem 2, Definition 1). We obtain the fact that a Fano 3-fold with the maximal basis-dimension 2 admits a projective bundle over a non-singular surface as a $\mathbf{Q}$-factorial terminal modification (Theorem 3). We try to make clear the stucture of a Fano 3-fold in this family: what kind of surface occurs as a basis, what kind of projective bundle appears as a $\mathbf{Q}$-factorial terminal modification and which parts on the projective bundle are contracted in a Fano 3-fold.

The author would like to thank Professors Nakayama and Kei-ichi Watanabe and also other members of Waseda Seminar for their stimulating discussion during the preparation of this article. In particular Nakayama's proof of Proposition 1 helped her very much and also K-i. Watanabe's comment "a Weil divisor on a $\mathbf{Q}$-factorial terminal singularity is Cohen-Macaulay" was very helpful in the proof of Theorem 2.

1. The case $\dim\Sigma = 0$

THEOREM 1A([I]). *Let X be a Fano 3-fold with $\dim\Sigma = 0$. Then there exist a normal surface S which is either an Abelian surface or a normal K3-surface and an ample invertible sheaf $\mathcal{L}$ on S such that X is the contraction of the negative section of a projective bundle $\mathbf{P}(\mathcal{O}_S \oplus \mathcal{L})$. Here a normal K3-surface implies a normal projective surface with the trivial canonical sheaf and has only rational singularities.*

THEOREM 1B([I]). *Let X be a projective cone over a surface S which is either an*

Received May 15, 1993.