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FACTORED ARRANGEMENTS OF HYPERPLANES

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Let $\mathbf{K}$ be a field and let V be a vector space over $\mathbf{K}$. An *arrangement of hyperplanes* in V is a finite family $\mathcal{A}$ of hyperplanes of V through the origin. An arrangement $\mathcal{A}$ of hyperplanes is said to be *real* (resp. *complex*) if $\mathbf{K} = \mathbf{R}$ is the field of real numbers (resp. if $\mathbf{K} = \mathbf{C}$ is the field of complex numbers).

With an arrangement $\mathcal{A}$ of hyperplanes, one can associate a graded torsion-free $\mathbf{Z}$-algebra $A(\mathcal{A})$, called the *Orlik-Solomon algebra* of $\mathcal{A}$. If $\mathcal{A}$ is a complex arrangement, then $A(\mathcal{A})$ is isomorphic to the cohomology algebra of the *complement*

$$M(\mathcal{A}) = V - (\cup_{H\in\mathcal{A}} H)$$

of $\mathcal{A}$ (see [OS1]). The *Poincaré polynomial* $\mathrm{Poin}(\mathcal{A}, t)$ of $\mathcal{A}$ is the Poincaré polynomial of $A(\mathcal{A})$, namely,

$$\mathrm{Poin}(\mathcal{A}, t) = \sum_{n=0}^{\infty} \dim(A^n(\mathcal{A}))t^n .$$

We refer to [Or] and [OT] for good expositions on the theory of arrangements of hyperplanes and, more precisely, on Orlik-Solomon algebras.

Let $\mathcal{A}$ be a real arrangement of hyperplanes. A *chamber* of $\mathcal{A}$ is a connected component of $V - (\cup_{H\in\mathcal{A}} H)$. We denote by $\mathcal{C}(\mathcal{A})$ the set of chambers of $\mathcal{A}$. For $C, D \in \mathcal{C}(\mathcal{A})$, we denote by $\mathcal{S}(C, D)$ the set of hyperplanes of $\mathcal{A}$ which separate C and D. For a fixed chamber $C_0 \in \mathcal{C}(\mathcal{A})$, we partially order $\mathcal{C}(\mathcal{A})$ by

$$C \leq D \quad \text{if} \quad \mathcal{S}(C_0, C) \subseteq \mathcal{S}(C_0, D) .$$

$\mathcal{C}(\mathcal{A})$ provided with this order is denoted by $P(\mathcal{A}, C_0)$. It is a ranked bounded poset of finite rank, where $\mathrm{rank}(C) = |\mathcal{S}(C_0, C)|$ for $C \in \mathcal{C}(\mathcal{A})$. Its smallest element is C_0 and its greatest one is the chamber $-C_0$ opposite to C_0. The *rank-generating function* of $P(\mathcal{A}, C_0)$ is

$$\zeta(P(\mathcal{A}, C_0), t) = \sum_{C\in\mathcal{C}(\mathcal{A})} t^{\mathrm{rank}(C)} .$$

The poset $P(\mathcal{A}, C_0)$ has been introduced and investigated by Björner, Edelman and Ziegler [Ed] [BEZ].

Let $\mathcal{A}$ be a real arrangement of hyperplanes. If $\mathcal{A}$ is either a supersolvable arrangement or a Coxeter arrangement, then there exist some integers $b_1, \ldots, b_l$ and a chamber $C_0 \in \mathcal{C}(\mathcal{A})$ such that the Poincaré polynomial of $\mathcal{A}$ factors as

$$(1) \qquad \mathrm{Poin}(\mathcal{A}, t) = \prod_{i=1}^{l} (1 + b_i t)$$

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