T. URABE
KODAI MATH. J.
17 (1994), 395—401

DYNKIN GRAPHS AND TRIANGLE SINGULARITIES

BY TOHSUKE URABE

In Arnold's classification list of singularities (Arnold [1].) we find interesting singularities to be studied. Though we find singularities of any dimension in Arnold's list, we consider singularities of dimension two in particular. Among them there is a class called exceptional singularities or triangle singularities. This class consists of fourteen singularities. It is known that they are closely related to K3 surfaces with the structure of elliptic surfaces. (Looijenga [4].) Here we would like to consider the following nine singularities of these fourteen ones:

$$E_{12},\ Z_{11},\ Q_{10},$$
$$E_{13},\ Z_{12},\ Q_{11},$$
$$E_{14},\ Z_{13},\ Q_{12}.$$

(The remaining five triangle singularities are W_{12}, W_{13}, S_{11}, S_{12} and U_{12}.) We assume that the ground field is the complex field $\mathbf{C}$.

Recall here that a connected Dynkin graph of type A, D or E corresponds to a surface singularity called a rational double point. Let Ξ be a class of surface singularities. By $PC(\Xi)$ we denote the set of Dynkin graphs Γ with several components such that there exists a small deformation fiber Y of a singularity belonging to Ξ satisfying the following conditions:

1. Y has only rational double points as singularities.
2. The combination of rational double points on Y corresponds exactly to Γ. (The type of each component of Γ corresponds to the type of the singularity on Y and the number of components of each type corresponds to the number of singularities of each type on Y.)

Note that by definition every graph in $PC(\Xi)$ has only components of type A, D or E. We would like to study $PC(\Xi)$ for $\Xi = E_{12},\ Z_{11},\ \dots,\ Q_{12}$

THEOREM. *Let Ξ be one of the above nine classes of singularities. The following two conditions are equivalent.*

(A) $\Gamma \in PC(\Xi)$.

(B) *The Dynkin graph Γ has only components of type A, D or E, and can be made from the essential basic graph depending on Ξ by a combination of two of elementary transformations and tie transformations.*

1991 Mathematics Subject Classification. 14B12, 14J17, 32S30
Received May 15, 1993.