J.-T. LU
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HYPERSURFACES OF A SPHERE WITH 3-TYPE QUADRIC REPRESENTATION

BY JI-TAN LU

Abstract

We study hypersurfaces of a sphere with 3-type quadric representation. Two theorems are obtained, and some eigenvalue inequalities are proved.

0. Introduction

Let $\Phi: M^n \to E^m$ be an isometric immersion of an n-dimensional compact Riemannian manifold into the Euclidean space, Δ and $\text{spec}(M^n)=\{0<\lambda_1<\lambda_2<\cdots \nearrow +\infty\}$ be the Laplacian and the spectrum of M^n, respectively. Then we have the decomposition $\Phi=\sum_{u\geq 0}\Phi_u$, $u\in N$, where $\Phi_u: M^n \to E^m$ is a differentiable mapping such that $\Delta\Phi_u=\lambda_u\Phi_u$, moreover Φ_0 is a constant mapping (it is the center of mass of M^n). M^n is said to be of finite type if the decomposition consists of only a finite number of non-zero terms, and of k-type if there are exactly k non-zero Φ_u's$(\Phi_{u_1}, \cdots, \Phi_{u_k})$ in the decomposition. In the latter case, we also call the immersion Φ to be of k-type.

Finite type submanifolds of a hypersphere $S^{m-1}\subset R^m$ have been studied by many authors. For example, see [5], [2], [9], [3]. In [5] mass-symmetric 2-type hypersurfaces of S^{m-1} were characterized. In [2] it was proved that a compact 2-type hypersurface of S^{m-1} is mass-symmetric if and only if it has constant mean curvature. In [9] Nagatomo showed that many 2-type hypersurfaces of a hypersphere are mass-symmetric and that there is no compact hypersurface of constant mean curvature in a hypersphere which is of 3-type. In particular, Barros and Garay [3] proved that the Riemannian product of two plane circles of different and stuitable radii is the only 2-type surfaces in $S^3\subset R^4$.

On the other hand, let $\Psi: M^n \to S^{n+p}(1)$ be a minimal isometric immersion of an n-dimensional compact Riemannian manifold into the unit sphere, $SM(n+p+1)=\{P\in gl(n+p+1, R) \mid P^t=P\}$, and $f: S^{n+p}(1)\to SM(n+p+1)$ be the order 2 immersion of $S^{n+p}(1)$. We consider the associated isometric immersion $\Phi=f\circ\Psi: M^n \to SM(n+p+1)$, which is called the quadric representation of M^n. In [8], Ros characterized minimal submanifolds in $S^{n+p}(1)$ with 2-type quadric representation. Later, Lu [7] proved that the Clifford torus $M_{m,m}$ are the only

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