D.J. HALLENBECK AND K. SAMOTIJ
KODAI MATH. J.
17 (1994), 273—289

ON THE SHARP GROWTH OF ANALYTIC CAUCHY-STIELTJES TRANSFORMS

By D. J. Hallenbeck and K. Samotij*

Introduction

Let $\Delta=\{z: |z|<1\}$ and $\Gamma=\{z: |z|=1\}$. Let $\mathcal{M}$ denote the set of complex-valued Borel measures on Γ. For each $\alpha\geqq 0$ the family $\mathcal{F}_\alpha$ of functions analytic in Δ is defined as follows. If $\alpha>0$ then $f\in\mathcal{F}_\alpha$ provided that there exists $\mu\in\mathcal{M}$ such that

$$(1)\qquad f(z)=f_\mu(z)=\int_\Gamma \frac{1}{(1-\bar{\zeta}z)^\alpha}\,d\mu(\zeta)$$

for $|z|<1$. Also, $f\in\mathcal{F}_0$ provided that there exists $\mu\in\mathcal{M}$ such that

$$(2)\qquad f(z)=f_\mu(z)=\int_\Gamma \log\frac{1}{(1-\bar{\zeta}z)}\,d\mu(\zeta)+f(0)$$

for $|z|<1$ (Here and throughout this paper every logarithm means the principal branch.). The classes $\mathcal{F}_\alpha$ for $\alpha\geqq 0$ were first studied in [3] and [4]. Of course, the case $\alpha=1$ is classical and well studied in the literature. The mapping from $\mathcal{M}$ to $\mathcal{F}_\alpha$ given by $\mu\to f_\mu$ is not one-to-one, i.e., the correspondence between measures and functions in $\mathcal{F}_\alpha$ is not unique. Suppose that $\mu\in\mathcal{M}$. Let $|\mu|$ denote the total variation norm of μ and let $\|\mu\|=|\mu|(\Gamma)$. For $|\zeta|=1$ and $0<x\leqq\pi$ let $I(\zeta, x)$ denote the closed arc on Γ centered at ζ and having length $2x$. A function w is defined on $[0, \pi]$ by

$$(3)\qquad w(x)=|\mu|(I(\zeta, x)) \quad \text{for} \quad 0<x\leqq\pi \quad \text{and} \quad w(0)=0.$$

To indicate the dependence of w on ζ and x we sometimes write $w(x)=w(x, \zeta, \mu)$ or $w(x)=w(x, \mu)$. As explained in [1] formula (1) is equivalent to

$$f(z)=\int_{-\pi}^{\pi} \frac{1}{(1-e^{-it}z)^\alpha}\,dg(t)$$

where g is a complex-valued function of bounded variation on $[-\pi, \pi]$. Similar remarks apply to (2). We point out, that in the standard way, our measures may be regarded as being defined on $[-\pi, \pi]$ rather than on Γ. This is noth-

* This research was partially supported by a grant from Komitet Badan Naukowych.
Received December 15, 1992; revised November 29, 1993.