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GEOMETRY AND TOPOLOGY OF SUBMANIFOLDS IMMERSED IN SPACE FORMS AND ELLIPSOIDS

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Abstract

Let M^m be a compact submanifold of a simply connected space form $N^n(c)$ with $c \geqq 0$. Denote by s and H the square length of the second fundamental form and the mean curvature vector field of M respectively. By introducing a selfadjoint linear operator Q^A associated with the shape operator of M, we show that there are no stable currents in M and topologically, M is a sphere if $s<H^2/(m-1)$. For an immersed submanifold of the ellipsoid we show that appropriate assumption on Q^A implies the vanishing of a given homology group.

1. Introduction

Let M^m be a submanifold immersed in a Riemannian manifold N^n. Denote by $V(N, M)$ the normal bundle of M in N. For a smooth section $\nu \in C(V(N, M))$, the shape operator A_ν determined by ν is given by

$$\langle A_\nu X, Y\rangle = \langle h(X, Y), \nu\rangle ,$$

where $X, Y \in C(TM)$ and h is the second fundamental form of M.

In 1973, by using techniques of the calculus of variations in geometric measure theory, H. B. Lawson and J. Simons [4] showed the following

THEOREM LS. *Let M^m be a compact submanifold of S^n and p a given integer, $p \in (0, m)$. If for any $x \in M$ and any orthonormal basis $\{e_i, e_\alpha\}$ $(i=1, \cdots, p;\ \alpha=p+1, \cdots, m)$ of T_xM the following condition is satisfied*

$$\sum_{i,\alpha} [2\|h(e_i, e_\alpha)\|^2 - \langle h(e_i, e_i), h(e_\alpha, e_\alpha)\rangle] < p(m-p),$$

then there is no stable p-current in M and hence

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