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SOME RESULTS ON RIGIDITY OF HOLOMORPHIC MAPPINGS

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1. In this paper we study rigidity properties of holomorphic mappings. Let X and Y be complex normed spaces. Let D_1 be a balanced domain in X and D_2 be a bounded convex balanced domain in Y. We consider holomorphic mappings f from D_1 into D_2. We prove two theorems. One of them is a generalization of the Schwarz lemma, which gives an upper bound for $\mu_{D_2}(f(x))$, $x \in D_1$. Here μ_{D_2} denotes the Minkowski functional of D_2. We also discuss the extremal mappings related to the Schwarz lemma. We deduce as a corollary the following fact: if $f: X \to Y$ is a holomorphic mapping which satisfies $\|f(x)\| = \|x\|$ for all $x \in X$, then f is linear. Another theorem gives a lower bound for $\mu_{D_2}(f(x))$, $x \in D_1$. Finally we are concerned with the limits of sequences of automorphisms of bounded domains. It is known that if D is a bounded domain in C^n and if a mapping $f: D \to D$ is a pointwise limit of a sequence of automorphisms of D, then f is also an automorphism of D. However, in the case that D is a bounded domain in a complex normed space X the limit $f: D \to D$ need not be an automorphism of D. We give a simple counterexample. Using the above two theorems we show that the limit f is one-to-one.

2. We summarize the main notation and terminology used in this paper. Let X be a complex normed space and let D be a domain in X. The Minkowski functional μ_D of D is defined by

$$\mu_D(x) = \inf\{t > 0 : t^{-1}x \in D\} \qquad (x \in X).$$

We denote the open ball with center at a and radius r in X by $B(a, r)$. Then we have that $\mu_{B(0,r)}(x) = r^{-1}\|x\|$.

Let X and Y be complex normed spaces and let D be a domain in X. A mapping $f: D \to Y$ is said to be holomorphic in D if, corresponding to every $a \in D$, there exist a power series $\sum_{k=0}^{\infty} P_k$ and a positive number ρ such that f is expressed by

$$f(x) = \sum_{k=0}^{\infty} P_k(x-a) \qquad (x \in B(a, \rho)).$$

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