H. TANEMURA
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HOMOGENIZATION OF A REFLECTING BARRIER BROWNIAN MOTION IN A CONTINUUM PERCOLATION CLUSTER IN R^d

Dedicated to Professor Hiroshi Tanaka on his 60th birthday

BY HIDEKI TANEMURA*

1. Introduction and statement of result

Let $\mathfrak{M}$ be the set of all countable subsets η of $\boldsymbol{R}^d$ ($d\geqq 2$) satisfying $N_K(\eta)<\infty$ for any compact subset K, where $N_A(\eta)$ is the number of points of η in $A\subset\boldsymbol{R}^d$. $\mathfrak{M}$ is equipped with $\mathfrak{B}(\mathfrak{M})$ the σ-field which is generated by $\{\eta\in\mathfrak{M}: N_A(\eta)=n\}$, $A\in\mathfrak{B}(\boldsymbol{R}^d)$, $n\in\boldsymbol{N}$. For $\eta\in\mathfrak{M}$, $r>0$ and two disjoint regions A_1 and A_2 in $\boldsymbol{R}^d$, we say that a continuous curve γ is an occupied (*resp.* a vacant) connection of A_1 and A_2 in a region A with respect to (η, r) if $\gamma\cap A_1\neq\emptyset$, $\gamma\cap A_2\neq\emptyset$, $\gamma\subset A$ and $\gamma\subset U_r(\eta)$ (*resp.* $\gamma\cap\overline{U_r(\eta)}=\emptyset$), where $U_r(\eta)$ stands for the r-neighborhood of η and $U_r(x)$ is the abbreviated form for $U_r(\{x\})$. A continuum percolation model is obtained if a distribution ν on the space $[0, \infty)\times\mathfrak{M}$ is given. In this paper we consider the case $\nu=\delta_r\otimes\mu_\lambda$, $r>0$, $\lambda>0$, where δ_r is the Dirac measure corresponding to the point r and μ_λ is a Poisson distribution on $\mathfrak{M}$ with intensity measure λdx, that is, for any disjoint system $\{A_1, A_2, \cdots, A_m\}\subset\mathfrak{B}(\boldsymbol{R}^d)$ such that $|A_i|=\int_{A_i}dx<\infty$, $i=1, 2, \dots, m$, $N_{A_1}(\eta), \dots, N_{A_m}(\eta)$ are independent random variables on the probability space $(\mathfrak{M}, \mathfrak{B}(\mathfrak{M}), \mu_\lambda)$ and

$$\mu_\lambda(N_{A_i}=n)=\frac{(\lambda|A_i|)^n}{n!}\exp(-\lambda|A_i|), \qquad i=1, 2, \dots, m, \ n\in\boldsymbol{N}\cup\{0\}.$$

This percolation model is called the 'Poisson blob model'. (See Grimmett [9].) It should be viewed as a continuum analogue of the discrete site percolation model. Instead of sites being independently occupied we have a Poisson process on $\boldsymbol{R}^d$ with each Poisson point being the center of an occupied ball of radius r. Now we define two regions in $\boldsymbol{R}^d$,

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