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KODAI MATH. J.
17 (1994), 69—81

EQUIVALENCE OF EULERIAN AND LAGRANGIAN WEAK SOLUTIONS OF THE COMPRESSIBLE EULER EQUATION WITH SPHERICAL SYMMETRY

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1. Introduction

One of the typical equations in fluid mechanics is the compressible Euler equation which describes the inviscid motion of an isentropic gas. The compressible Euler equation with an external force $\vec{f}$ in R^n is the $(n+1)\times(n+1)$ system of conservation laws,

$$\left\{\begin{array}{l}\rho_t+\sum_{j=1}^{n}\frac{\partial}{\partial x_j}(\rho u_j)=0,\\ (\rho u_i)_t+\sum_{j=1}^{n}\frac{\partial}{\partial x_j}(\rho u_i u_j+\delta_{ij}P)=\rho f_i, \quad (i=1, 2, \cdots, n)\end{array}\right. \tag{1.1}$$

where ρ is the density, $\vec{u}={}^t(u_1, u_2, \cdots, u_n)$ is the velocity, P is the scalar pressure with δ_{ij} the Kronecker delta and $\vec{f}(t, x)={}^t(f_1, f_2, \cdots, f_n)$ is the external force. For an isentropic gas P satisfies

$$P=a^2\rho^\gamma, \tag{1.2}$$

where $a>0$ and $\gamma\geqq 1$ are given constants.

Let us consider the initial and boundary value problem for (1.1) in $t\geqq 0$, $x\in\Omega\subset R^n$ with the following conditions.

$$\vec{u}(0, x)=\vec{u}_0(x), \qquad \rho(0, x)=\rho_0(x), \tag{1.3}$$

$$\vec{u}\cdot\vec{n}=0 \qquad \text{if } x\in\partial\Omega, \tag{1.4}$$

where $\vec{n}$ is the unit vector normal to the boundary.

$\vec{u}(t, x)$ and $\rho(t, x)$ are called weak solutions of (1.1), (1.3) and (1.4) if u_i, ρ $\in L^\infty((0, T)\times\Omega)$ $(i=1, 2, \cdots, n)$, $f_i(t, x)\in L^1_{loc}((0, T)\times\Omega)$ $(i=1, 2, \cdots, n)$ and if they satisfy the following $n+1$ integral identities

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Received August 19, 1993.