RIEMANNIAN SUBMERSION WITH ISOMETRIC REFLECTIONS WITH RESPECT TO THE FIBERS

Dedicated to Professor Yoji Hatakeyama on his sixtieth birthday

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1. Introduction

The concept of Riemannian submersion was introduced by O'Neil [10] and is discussed by him and others ([4], [8], etc). A Riemannian submersion with totally geodesic fibers often appears in the differential geometry.

On the other hand, in [3] Chen and Vanhecke introduced the notion of the reflections with respect to submanifolds. And there are some studies of reflections with respect to the fibers in a Riemannian submersion or local fibering of a Sasakian manifold (e.g. [2], [9], [11]).

In this paper, we shall consider a Riemannian submersion \( \pi : M \to N \) with fibers of dimension one. In Section 2, we give some properties of the integrability tensor \( A \) with respect to \( \pi \). In Section 3, we shall consider the isometric reflections with respect to the fibers in Riemannian submersion which satisfies certain conditions. Our result is a generalization of the result of Kato and Motomiya [6], [11]. And particularly, in the case of 3-dimension, we get the following result: the reflections with respect to the fibers are isometries if and only if \( M \) admits a Sasakian locally \( \phi \)-symmetric structure. Finally, we give a complete classification of 3-dimensional Riemannian manifolds with isometric reflections with respect to the fibers.

2. Riemannian submersion

In this section we collect some results on Riemannian submersions. Let \( \pi : M \to N \) be a Riemannian submersion. Let \( X \) denote a tangent vector at \( x \in M \). Then \( X \) decomposes as \( \mathcal{V}X + \mathcal{H}X \), where \( \mathcal{V}X \) is tangent to the fiber through \( x \) and \( \mathcal{H}X \) is perpendicular to it. If \( X = \mathcal{V}X \), \( X \) is called a vertical vector. If \( X = \mathcal{H}X \), it is called horizontal. Let \( \nabla \) and \( \hat{\nabla} \) denote the Riemannian connections of \( M \) and \( N \) respectively.

We define tensors \( T \) and \( A \) associated with the submersion by

\[
(1) \quad T_E F = \mathcal{V}\nabla_{\mathcal{H}}E F + \mathcal{H}\mathcal{V}\nabla_{\mathcal{H}}E F,
\]

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