S. MAEDA, Y. OHNITA AND S. UDAGAWA
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ON SLANT IMMERSIONS INTO KÄHLER MANIFOLDS

BY SADAHIRO MAEDA, YOSHIHIRO OHNITA AND SEIICHI UDAGAWA

Introduction.

Let $\varphi: M \to N$ be an isometric immersion of a Riemannian manifold into an almost Hermitian manifold with almost complex structure $\tilde{J}$. Then, φ is called *slant* if the angle between $\tilde{J}\varphi_*(X)$ and $\varphi_*(T_pM)$ is constant for any $X \in T_pM$ and any $p \in M$. The typical examples of slant immersions are Kähler immersions and totally real immersions, where the slant angles are 0 and $\pi/2$, respectively. A slant immersion is called *proper* if it is neither a Kähler immersion nor a totally real immersion. In the case where M is a Riemann surface and N is a Kähler manifold, the slant angle was introduced as the *Kähler angle* and studied by S. S. Chern and J. G. Wolfson [CW]. Examples of slant immersions of a Riemann sphere S^2 into a complex projective space $\boldsymbol{C}P^n$ were given as the Veronese sequence of harmonic maps from S^2, which are classified in [BO] and [BJRW] in the case where S^2 has constant curvature (see also [O1]). The present concept of slant immersion was first introduced and studied by B. Y. Chen [C]. The examples of proper slant immersions into $\boldsymbol{C}^4$ are given in [C-T]. Recently, Tazawa [T] has given examples of slant immersions into $\boldsymbol{C}^n$ with any given slant angle. However, there are a few results on slant submanifolds in $\boldsymbol{C}P^n$. In this case, any general method to check whether given an immersion is slant or not is not known.

The main purpose of this paper is to study slant submanifolds in $\boldsymbol{C}P^n$. In Section 1, we give some sufficient conditions for an isometric immersion φ of a compact Kähler manifold M into a Kähler manifold N to be slant (Theorem 1.2, Proposition 1.3). In Theorem 2.1 of Section 2, we show that the condition of Theorem 1.2 is satisfied for a G-equivariant isometric immersion of a Kähler C-space M with $b_2(M)=1$ into $\boldsymbol{C}P^n$. In this case, the slant angle is explicitly given by $\cos^{-1}(4\pi|\deg(\varphi)|/\tilde{c}\,\mathrm{vol}(S))$, where S is a rational curve of M which represents a positive generator of $\mathrm{H}_2(M;\boldsymbol{Z})$ and $\tilde{c}$ is a (constant) holomorphic sectional curvature of $\boldsymbol{C}P^n$. Consequently, it turns out that $\mathrm{SU}(m+1)$-equivariant isometric immersions of $\boldsymbol{C}P^m$ into $\boldsymbol{C}P^N$ constructed and treated by the first and second author ([M], [O2]) are slant, and that pluriharmonic maps constructed in [OU] give many examples of proper slant immersions into $\boldsymbol{C}P^n$.

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