M. OKA
KODAI MATH. J.
16 (1993), 181—195

FINITENESS OF FUNDAMENTAL GROUP OF COMPACT CONVEX INTEGRAL POLYHEDRA

BY MUTSUO OKA

§1. Introduction and statement of the result.

Let Δ_i, $i=1, \cdots, k$, be given compact convex integral polyhedra in $\boldsymbol{R}^m$. We consider the following integer "combinatorial connectivity" $\alpha(\Delta_1, \cdots, \Delta_k)$ which is defined in [Ok6] by

$$\alpha(\Delta_1, \cdots, \Delta_k)=\min\left\{\dim\left(\sum_{i\in I}\Delta_i\right)-|I|\,;\, I\subset\{1, \cdots, k\},\ I\neq\emptyset\right\}.$$

We assume that $\alpha(\Delta_1, \cdots, \Delta_k)\geqq 0$. For any integral covector P, we consider the restriction $P|_{\Delta_i}$ to Δ_i of the corresponding linear function associated with P. Let $\Delta(P;\Delta_i)$ be the face where $P|_{\Delta_i}$ takes its minimal value ([Ok5, 6]). We denote the lattice of the integral covectors by N. We define the subgroup $K(\Delta_1, \cdots, \Delta_k)$ of N by

$$K(\Delta_1, \cdots, \Delta_k)=\langle P\in N;\, \alpha(\Delta(P;\Delta_1), \cdots, \Delta(P;\Delta_k))\geqq 0\rangle.$$

Here $\langle P\in N;\, P\in S\rangle$ is the subgroup of N which is generated by the covectors P in S. We also define $\Pi_1(\Delta_1, \cdots, \Delta_k):=N/K(\Delta_1, \cdots, \Delta_k)$. We call $K(\Delta_1, \cdots, \Delta_k)$ (respectively $\Pi_1(\Delta_1, \cdots, \Delta_k)$) *the boundary lattice group* (resp. *the fundamental group*) of the k-ple of polyhedra $\{\Delta_1, \cdots, \Delta_k\}$. The purpose of this paper is to prove:

MAIN THEOREM (1.1). *The boundary lattice group $K(\Delta_1, \cdots, \Delta_k)$ has rank m if and only if $\alpha(\Delta_1, \cdots, \Delta_k)\geqq 1$.*

The geometric interpretation is as follows. Let $h_1(\boldsymbol{u}), \cdots, h_k(\boldsymbol{u})$ be Laurent polynomials such that the respective Newton polygon $\Delta(h_i)$ is equal to Δ_i, for $i=1, \cdots, k$. Let us consider the variety:

$$Z^*=\{\boldsymbol{u}\in \boldsymbol{C}^{*m};\, h_1(\boldsymbol{u})=\cdots=h_k(\boldsymbol{u})=0\}.$$

We can choose the coefficients of $h_1, \cdots, h_k$ so that Z^* is a non-degenerate complete intersection variety in the sense of [Kh1, 2, Ok4, 5]. See §4 for the existence of such Laurent polynomials $h_1(\boldsymbol{u}), \cdots, h_k(\boldsymbol{u})$. Z^* is non-empty if and

Received January 21, 1993.