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TORUS SUM FORMULA OF SIMPLE INVARIANTS FOR 4-MANIFOLDS

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1. Introduction.

The topology of moduli spaces of anti-self-dual (ASD) connections is closely related with differentiable structures on 4-manifolds. In his celebrated paper [6], Donaldson has defined the polynomial invariants to distinguish differentiable structures on a 4-manifold. Though a significant result on the vanishing has been obtained at the same time, these invariants remain to be very difficult to determine completely. In fact, many examples have been calculated using an identification of irreducible ASD connections and stable vector bundles by Donaldson ([6], [7], [9], [12], [18]). But there are another direct approaches in the case that the dimension of ASD moduli is zero and that the invariant is just a number of the points in the moduli. For example Gompf and Mrowka have defined an invariant for 4-manifolds with torus end, using 0 or 1 dimensional ASD moduli, and proved that the invariant of the glued manifolds with solid torus can be emerged as the number of ASD connections which can be extended to over the solid torus. From a topological argument on K3 surface with elliptic fibration, they calculated the above numbers for fake K3 surfaces obtained by performing logarithmic transformations on embedded 2-tori. After that, Kromheimer has observed that the ASD moduli of Kummer surface comes down to the flat moduli as all (-2) curves tends to infinity, so the invariant could be computed algebraically [13]. The invariant obtained by 0-dimensional ASD moduli is said to be a simple invariant.

In this paper we give a torus sum formula of simple invariants for 4-manifolds. Our idea and formula are simple. Suppose that we have two simply connected closed 4-manifolds which contain a 2-torus with the trivial normal bundle. We assume that the complements are simply connected and the second Stiefel-Whitney class to define the $SO(3)$ bundle does not vanish on the 2-torus. Then any ASD connection converges to some ASD connections as the 2-torus tends to infinity. On the other hand, any ASD connection over the new 4-manifold obtained by torus sum also converges to some ASD connections as the bi-collar of the intermediate 3-torus is stretched to infinity. Hence we prove that the simple invariant of the new 4-manifold is the product of that of the

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