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THE EUCLIDEAN, HYPERBOLIC, AND SPHERICAL SPANS OF AN OPEN RIEMANN SURFACE OF LOW GENUS AND THE RELATED AREA THEOREMS

Dedicated to Professor Nobuyuki Suita on his sixtieth birthday

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Introduction.

It was almost fifty years ago that M. Schiffer [11] introduced the notion of *span* to study the theory of conformal mapping—or the theory of univalent functions if one would like to call it—of multiply connected plane domains along the line of Grötzsch and Grunsky. The notion has since been playing an important role in the theory of conformal mapping of planar Riemann surfaces. To deal with nonplanar Riemann surfaces equally, we shall have to take account of holomorphic *mappings* (into other Riemann surfaces) as well as holomorphic *functions,* and also have to generalize the notion of span. While the study of holomorphic mappings in its full generality is still immature, the theory of conformal embeddings (=injective holomorphic mappings) suffices for our purposes. More specifically, if we confine ourselves to those mappings which embed an open Riemann surface of finite genus into closed ones of the same genus, considerably satisfactory results could be expected. We have shown some of them in the preceding papers [12]-[15], on which the present article is based.

By the phrase *"of low genus"* we mean "either *of genus zero* or *of genus one*". We first consider the case of genus one. To state the preparatory facts briefly and clearly, it is convenient to introduce the term *"an open torus"*, which simply means an open Riemann surface of genus one. Meanwhile we keep the classical terminology *"a torus"* means a closed Riemann surface of genus one as usual. Sometimes the term *"a closed torus"* will be also used for the same purpose. A *compact continuation* of an open torus is, roughly speaking, a conformal embedding of the open torus into a closed torus which induces the prescribed correspondence between their canonical homology bases. We have shown in [13], among other things, that the set of moduli of the compact continuations of an open torus is a closed disk in the upper half plane, and that the diameter of this moduli disk gives a close analogue of Schiffer's span. Although the present work has been motivated by the investigation of open tori, the method does work, in principle, also for planar Riemann surfaces and yields new results

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